Everyday Nathematics® Student Math Journal 2

The University of Chicago School Mathematics Project



UCSMP Elementary Materials Component

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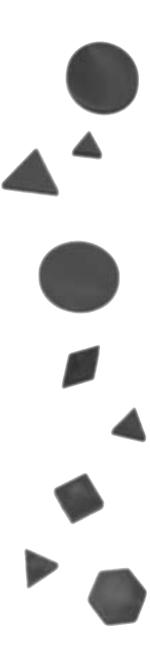
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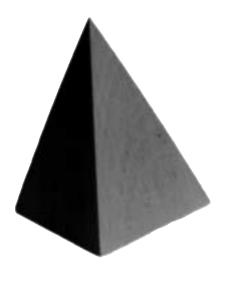
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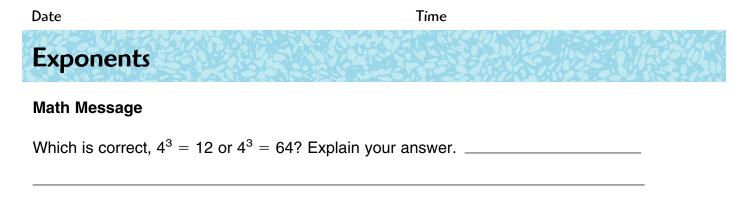
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Exponential Notation

In exponential notation, the **exponent** tells how many times the **base** is used as a factor. For example, $4^3 = 4 * 4 * 4 = 64$. The base is 4 and the exponent is 3.

1. Complete the table.

Exponential Notation	Base	Exponent	Repeated Factors	Product
54	5	4	5 * 5 * 5 * 5	625
	2	3		
			6 * 6 * 6 * 6	
			9 * 9	
			1 * 1 * 1 * 1 * 1 * 1 * 1	
	2			32

The Powers Key \frown

 Use your calculator to enter the keystrokes shown in the first column of the table. Record the calculator display in the second column.

Study your results. What is the function of the key?

Calculator Entry	Calculator Display
4 🔨 3 🖽	
3 🔨 2 🖽	
5 \land 3 🖽	

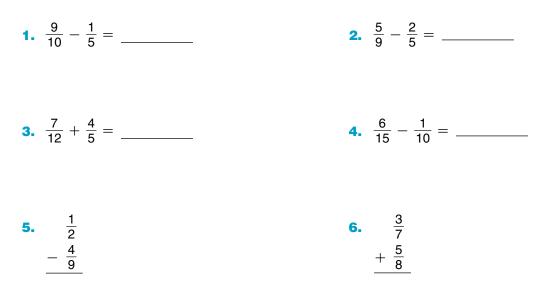
Date

Time

Exponents (cont.)	
Each problem below has a mistake. Find the mistake and tell what it is. Then solve the problem.	
3. $5^2 = 5 * 2 = 10$	
Mistake:	
Correct solution:	
4. $6^3 = 3 * 3 * 3 * 3 * 3 * 3 = 729$	
Mistake:	-
Correct solution:	-
5. $10^4 = 10 + 10 + 10 + 10 = 40$	
Mistake:	
Correct solution:	
Use your calculator to write the following numbers in standard notation.	
6. 7 * 7 * 7 * 7 = 7. 15 * 15 * 15 * 15 =	
8. $6^9 = $ 9. $5^8 = $	
10. $2^{12} =$ 11. 4 to the fifth power =	
Write <, >, or =.	
12. 10^2 2^{10} 13. 3^4 9^2 <i>Reminder:</i>	
14. 1^2 1^5 15. 5^4 500 > means <i>is greate</i>	r than.

Addition and Subtraction of Fractions

Find a common denominator. Then add or subtract.



Solve.

7. Regina is baking two different kinds of chocolate-chip cookies. One recipe calls for $\frac{1}{4}$ cup of chocolate chips. The other calls for $\frac{3}{4}$ cup of chocolate chips. How many cups of chocolate chips does she need in all?

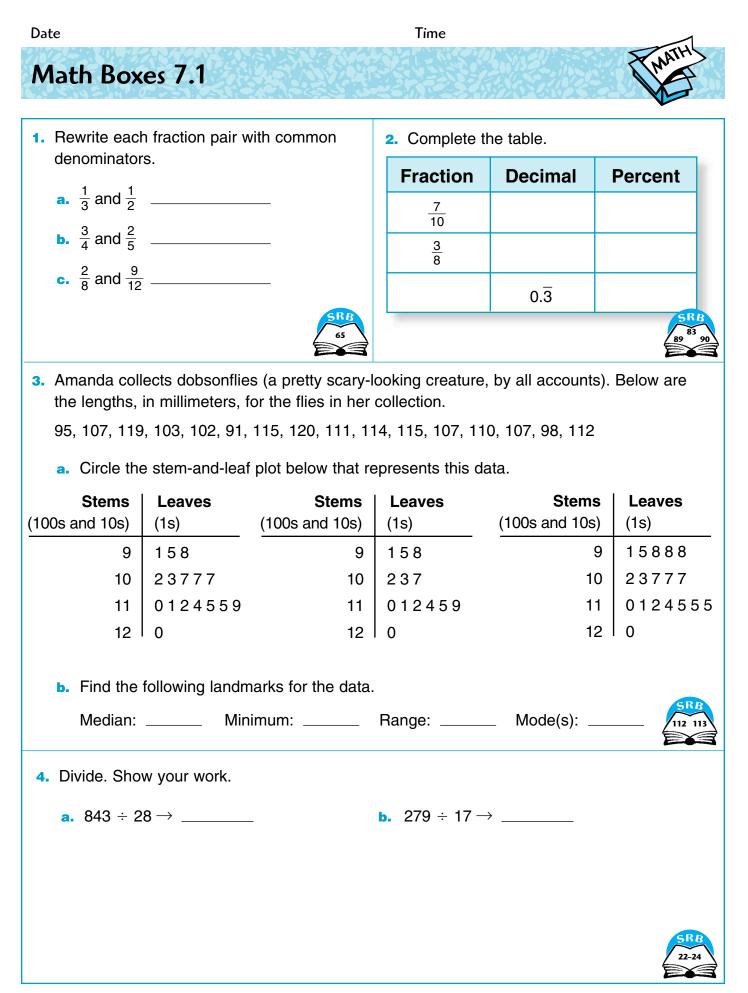
Write a number model:	

Solution: _____ cup

8. Roger found a long piece of heavy rope that was $24\frac{3}{4}$ feet long. It was a perfect rope for making jump ropes. If each jump rope is $8\frac{1}{4}$ feet long, how many can he make?

_____ jump ropes

Explain how you found your answer.



Use with Lesson 7.1.

Guides for Powers of 10

Study the place-value chart below.

Millions			Thousands			Ones			
Billions	Hundred- millions	Ten- millions	Millions	Hundred- thousands	Ten- thousands	Thousands	Hundreds	Tens	Ones
109	/0 ⁸	107	106	/0 ⁵	104	/0 ³	/0 ²	101	100

In our place-value system, the powers of 10 are grouped into sets of three: ones, thousands, millions, billions, and so on. These groupings are helpful for working with large numbers. When we write large numbers, we separate these groups of three with commas.

We have prefixes for these groupings and for other important powers of 10. You know some of these prefixes from your work with the metric system. For example, the prefix *kilo*- in *kilometer* identifies a kilometer as 1,000 meters. It is helpful to memorize the exponential notation and the prefixes for one thousand, one million, one billion, and one trillion.

Use the place-value chart for large numbers and the prefixes chart to complete the following statements.

Prefixes				
tera-	trillion (10 ¹²)			
giga-	billion (10 ⁹)			
mega-	million (10 ⁶)			
kilo-	thousand (10 ³)			
hecto-	hundred (10 ²)			
deca-	ten (10 ¹)			
uni-	one (10 ⁰)			
deci-	tenth (10 ⁻¹)			
centi-	hundredth (10 ⁻²)			
milli-	thousandth (10^{-3})			
micro-	millionth (10 ⁻⁶)			
nano-	billionth (10 ⁻⁹)			

Example

1 kilogram equals 10 or one <u>thousand</u> grams.

- 1. The distance from Chicago to New Orleans is about 10³ or one ______ miles.
- 2. A millionaire has at least 10 dollars.
- A computer with 1 megabyte of RAM memory can hold approximately 10¹ or one ______ bytes of information.
- A computer with a 1 gigabyte hard drive can store approximately 10¹ or one ______ bytes of information.
- According to some scientists, the hearts of most mammals will beat about 10⁹ or one ______ times in a lifetime.

Date

Negative Powers of 10

Our base-ten place-value system works for decimals as well as for whole numbers.

Tens	Ones	•	Tenths	Hundredths	Thousandths
10s	1s		0.1s	0.01s	0.001s

Negative powers of 10 can be used to name decimal places.

Example $10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{10} \times \frac{1}{10} = 0.1 \times 0.1 = 0.01$

Very small decimals can be hard to read in standard notation, so people often use number-and-word notation, exponential notation, or prefixes instead.

Guides for Small Numbers						
Number-and-Word Notation	Exponential Notation	Standard Notation	Prefix			
1 tenth	$10^{-1} = \frac{1}{10}$	0.1	deci-			
1 hundredth	$10^{-2} = \frac{1}{10 * 10}$	0.01	centi-			
1 thousandth	$10^{-3} = \frac{1}{10 * 10 * 10}$	0.001	milli-			
1 millionth	$10^{-6} = \frac{1}{10 * 10 * 10 * 10 * 10 * 10}$	0.000001	micro-			
1 billionth	$10^{-9} = \frac{1}{10 * 10 * 10 * 10 * 10 * 10 * 10 * 10 *$	0.00000001	nano-			
1 trillionth	$10^{-12} = \frac{1}{10*10*10*10*10*10*10*10*10*10*10*10*10*1$	0.000000000001	pico-			

Use the table above to complete the following statements.

- A fly can beat its wings once every 10⁻³ seconds, or once every one thousandth of a second. This is a ______ second.
- Earth travels around the sun at a speed of about 1 inch per microsecond.
 This is 10 second, or a second.
- Electricity can travel one foot in a nanosecond, or one ______ of a second. This is 10^{______} second.
- **4.** In 10^{1} second, or one picosecond, an air molecule can spin once.

This is a _____ of a second.

Date			Time					
Math Boxes 7	.2							MATH
1. Multiply. Use the particular	artial-products algorithm	n.						
a. 87 <u>* 65</u>	b. 39 <u>* 24</u>			С	99 <u>* 26</u>			
								SRB 19
2. Tell whether each r	number is prime or com	iposite.						
a. Number of hour	-							
	utes in $\frac{1}{12}$ of an hour							
c. Number of wee								
d. Number of mon								
e. Number of days	$\frac{3}{7}$ of a week							SRB 12 74
3. What is the measu	re of angle <i>T</i> ? P		Circle to $\frac{2}{3}$.	the fra	actions	that are	equiva	alent
4 T	5° A		<u>8</u> 9	<u>20</u> 30	<u>14</u> 21	<u>6</u> 10	<u>12</u> 18	
measure $\angle T = _$	0 							59-61

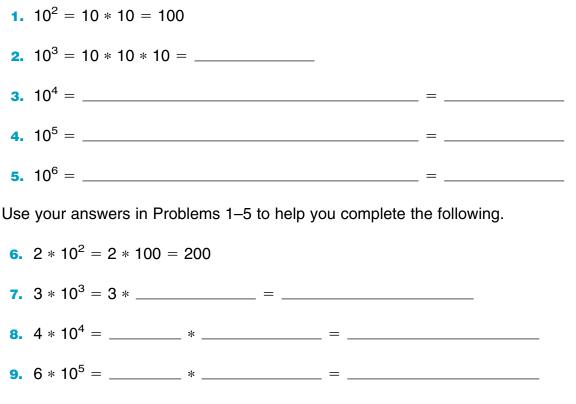
Use with Lesson 7.2.

Date

Time

Scientific Notation

Complete the following pattern.

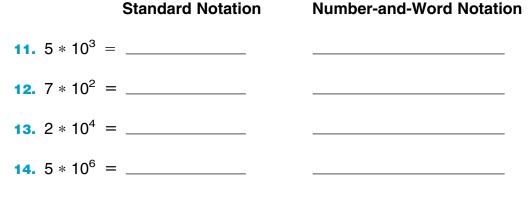


10. $8 * 10^6 =$ _____ * ____ = ____

Numbers written as the product of a number and a power of 10 are said to be in **scientific notation.** Scientific notation is a useful way of writing large or small numbers. Many calculators display numbers one billion or larger with scientific notation.

Example In scientific notation, 4,000 is written as $4 * 10^3$. It is read as "four times ten to the third power."

Write each of the following in standard notation and number-and-word notation.



Date Time Math Boxes 7.3 **1. a.** What is the perimeter of the rectangle? 2. a. Find an object in the room that is about 15 centimeters long. 12 units 8 units **b.** Find an object in the room that is about 3 inches long. **b.** What is the area? 3. a. Draw a circle that has a diameter of 4. Use a calculator to rename each of the 4 centimeters. following in standard notation. **a.** $3^{10} =$ _____ **b.** $8^4 =$ _____ **c.** $4^8 =$ _____ **d.** $5^7 =$ _____ **b.** The radius of the circle is **e.** $9^8 =$ _____ cm. 5. Solve. Do not use a calculator. **b.** 712 + 504 =_____ **a.** 287 + 395 = _____ **c.** 776 + _____ = 1,943 **d.** 2,080 = 948 +_____ **e.** _____ + 286 = 345

Date

History of the Earth

Geologists, anthropologists, paleontologists, and other scholars often estimate when important events occurred in the history of the Earth. For example, when did dinosaurs become extinct? When did the Rocky Mountains develop? The estimates are very broad, partly because events like these lasted for many years, and partly because dating methods cannot precisely pinpoint exact times so long ago.

Scientists base their estimates on the geological record—rocks, fossils, and other clues—and on the bones and tools left by humans long ago. Below is a list of events prepared by one group of scientists. All the data are approximations, and different estimates are given by other scientists.

Use the place-value chart on the next page to help you write, in standard notation, how long ago the events below took place.

<i>Example</i> Earth was formed about $5 * 10^9$ years ago. Find 10^9 on the place-value chart and
write 5 beneath it, followed by zeros in the cells to the right. Then use the chart to help
you read the number: $5 * 10^9 = 5$ billion.

What happened:	Some scientists say it happened about this many years ago:
1. Earth was formed.	5 * 10 ⁹ years
2. The first signs of life (bacteria cells) appeared.	4 * 10 ⁹
3. Fish appeared.	4 * 10 ⁸
4. Forests, swamps, insects, and reptiles appeared.	3 * 10 ⁸
5. Tyrannosaurus Rex lived; modern trees appeared.	1 * 10 ⁸
6. The first known human-like primates appeared.	6 * 10 ⁶
7. Woolly mammoths and other large ice-age mammals appeared.	8 * 10 ⁵
8. Humans first moved from Asia to North America.	2 * 10 ⁴

Challenge

 The first dinosaurs appeared; the Appalachian 	
Mountains formed.	2.5 * 10 ⁸
10. Dinosaurs became extinct.	6.5 * 10 ⁷

Source: The Handy Science Answer Book

History of the Earth (cont.)

	Billion	100 M	10 M	Million	100 Th	10 Th	Thousand	100	10	One
	10 ⁹	10 ⁸	10 ⁷	10 ⁶	10 ⁵	10 ⁴	10 ³	10 ²	10 ¹	10 ⁰
1.										
2.										
3.										
4.										
5.										
6.										
7.										
8.										
9.										
10.										

Reminder: Powers are calculated before other factors are multiplied. $5.5 * 10^4 = 5.5 * 10,000 = 55,000$.



That's a Yotta Numbers

Prefixes for very large and very small numbers, such as *tera*- (10^{12}) and *pico*- (10^{-12}) , were adopted by an international scientific and mathematics committee in 1960. Since then, scientists and mathematicians have routinely worked with still larger and smaller numbers and have updated the list of prefixes.

The most recent adoption was yotta- (10^{24}) in 1991. Yotta- is based on the Latin word for *eight*, because 1 septillion is equal to 1,000 to the eighth power (1,000⁸). Can you think of a prefix for 1 octillion and 1 nonillion?

Number	Exponential Notation	Prefix
1 quadrillion	10 ¹⁵	peta-
1 quintillion	10 ¹⁸	exa-
1 sextillion	10 ²¹	zetta-
1 septillion	10 ²⁴	yotta-
1 octillion	10 ²⁷	
1 nonillion	10 ³⁰	



Work with a partner to answer the following questions.

- **11.** According to the estimates of scientists, about how many years passed from the formation of Earth until the first signs of life?
- **12.** About how many years passed between the appearance of the first fish and the appearance of forests and swamps?
- 13. Make up and answer one or two questions of your own about data in the table of

Earth's history			

Challenge

14. According to the geological record, about how long did dinosaurs roam on Earth?

Date Time Math Boxes 7.4 1. Rewrite each fraction pair with common 2. Complete the table. denominators. Fraction Decimal Percent **a.** $\frac{2}{3}$ and $\frac{3}{5}$ _____ <u>2</u> 3 **b.** $\frac{3}{7}$ and $\frac{9}{10}$ _____ 0.95 **c.** $\frac{3}{8}$ and $\frac{18}{24}$ ______ 43% 3 5 0.8 3. a. Make a stem-and-leaf plot of the hand-span measures in Ms. Grip's fifth grade class. 163, 179, 170, 165, 182, 157, 154, 165, 170, 175, 162, 185, 158, 158, 170, 165, 162, 154 **b.** Find the following landmarks for the data.

Median: _____ Minimum: _____

Range: _____ Mode(s): _____

4. Divide. Show your work.

a. $21)\overline{493}$ _____ **b.** 35)623



Parentheses and Number Stories

Math Message

1. Make a true sentence by filling in the missing number.

a. 7.3 – (2.2 + 1.1) =	b. $(7.3 - 2.2) + 1.1 = $
c. 2.0 * (7.5 + 1.5) =	d. $(2.0 * 7.5) + 1.5 = $

2. Solve the following problem to get as many different answers as possible. Write a number sentence for each way.

$$6 * 4 - 2 / 2 = ?$$

Match each number story with the expression that fits that story.

3. Story 1Tom's total number of soda cans:Tom had 4 cans of soda.
He went shopping and bought
3 six-packs of soda cans.(4 + 3) * 6Story 2Tom had 4 six-packs of soda cans.
He went shopping and bought 3 more
six-packs of soda cans.4 + (3 * 6)

Parentheses and Number Stories (cont.)

4. Story 1

Alice ate 3 cookies before going to a party. At the party, Alice and 4 friends ate equal shares of 45 cookies.

Story 2

There was a full bag with 45 cookies, and an opened bag with 3 cookies. Alice and 4 friends ate equal shares of all these cookies.

5. Story 1

Mr. Chung baked 5 batches of cookies. Each of the first 4 batches contained 15 cookies. The final batch contained only 5 cookies.

Story 2

In the morning, Mr. Chung baked 4 batches of 15 cookies each. In the afternoon, he baked 5 more batches of 15 cookies each.

6. A grocery store received a shipment of 120 cases of apple juice. Each case contained 4 six-packs of cans. After inspection, the store found that 9 cans were damaged.

Write an expression that represents the number of undamaged cans.

Number of cookies Alice ate:

3 + (45 / 5)

(3 + 45) / 5

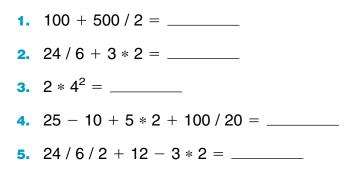
Number of cookies baked:

(15 * 4) + 5

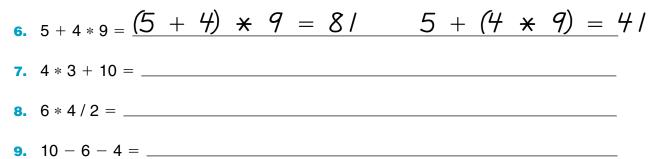
Date

Order of Operations

Use the rules of order of operations to complete these number sentences.



Insert parentheses in each problem below to get as many different answers as you can. The first one is done as an example.



First, solve these problems by hand. Then solve them with your calculator.

	By Hand	Calculator
10.	5 + 3 * 6 =	5 + 3 * 6 =
11.	3 * 6 + 5 =	3 * 6 + 5 =
12.	36 - 18 / 6 =	36 - 18 / 6 =
13.	44 - 6 * 5 =	44 - 6 * 5 =

- 14. a. Does your calculator obey the correct order of operations? _____
 - **b.** If your calculator obeys the correct order of operations, how do you know?
 - c. If your calculator doesn't obey the correct order of operations, then what order does it use?

American Tour: Inequalities

Use the American Tour section of your *Student Reference Book* to make comparisons of population, geographic area, or other data. Use > or < to write an inequality for each comparison.

Symbol	Meaning
>	is greater than
<	is less than

	Comparison	Inequality
1.		
2.		
з.		
4.		
5.		
6.		
7.		

Date				Time				
Math Boxe	s 7.5							MATH
1. Multiply. Use t	he partial-produc	ts algorithm.						
a. 43 <u>* 78</u>	b.	19 <u>* 86</u>				79 <u>42</u>		
b. Number of	each number is pr millimeters in 2.9 inches in $1\frac{1}{2}$ yard centimeters in 0.) centimeters ds						
d. Number of	inches in $\frac{5}{6}$ foot feet in $4\frac{1}{3}$ yards							
3. What is the m	easure of angle <i>I</i>	₽?	4.	Circle to $\frac{3}{8}$.	the fract	ions that	t are equ	uivalent
27° F	20°	S		<u>6</u> 12	<u>9</u> 24	<u>8</u> 3	<u>4</u> 9	<u>15</u> 40
measure ∠ <i>R</i>	=	o 						

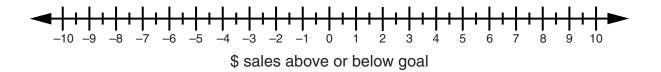
Positive and Negative Numbers on a Number Line

- 1. Graph each of the following bicycle race events on the number line below. Label each event with its letter. (*Hint:* Zero on the number line stands for the starting time of the race.)
 - A Check in 5 minutes before the race starts.
 - **C** Get on the bicycle 30 seconds before the race starts.
 - E Complete the first lap 3 minutes, 15 seconds after the race starts.
- **B** Change gears 30 seconds after the race starts.
- **D** The winner finishes at 6 minutes, 45 seconds.
- **F** Check the tires 2 minutes before the race starts.



2. Mr. Pima's class planned a raffle. Five students were asked to sell raffle tickets. The goal for each student was \$50 in ticket sales. The table below shows how well each of the five students did. Complete the table. Then graph the amounts from the last column on the number line below the table. Label each amount with that student's letter.

Student	Ticket Sales	Amount That Ticket Sales Were Above or Below Goal
А	\$5.50 short of goal	-\$5.50
В	Met goal exactly	
С	Exceeded goal by \$1.75	
D	Sold \$41.75	
E	Sold \$53.25	



Use with Lesson 7.6.

Answer the following. 7. What is the value of π to two decimal places?

8. $-\pi =$ _____

List the numbers in order from least to greatest.

9. $-10, 14, -100, \frac{8}{2}, -17, 0$ ______

10. -0.5, 0, -4, -π, -4.5 _____

12. Name four negative numbers greater than $-\pi$.

Answer the following.

230

11. Name four positive numbers less than π .

Write > or <.

1. -5 _____ 5

4. 14 _____ 7

We use the > (greater than) symbol to write +10 > +5.

+10 is greater than +5, because +10 is to the right of +5.

+ + +

-10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4

Reminder: When writing the > or <symbol, be sure the arrow tip points to the smaller number.

9 10

Comparing and Ordering Numbers

-10 is less than -5, because -10 is to the left of -5.

We use the < (less than) symbol to write -10 < -5.

+

+

For any pair of numbers on the number line, the number to the left is less than the number to the right.

> ╋ ╋

Time

2. 10 _____ -10 **3.** -10 _____ 0

5. -14 _____ -7 **6.** 0 _____ $-6\frac{1}{2}$

5

6

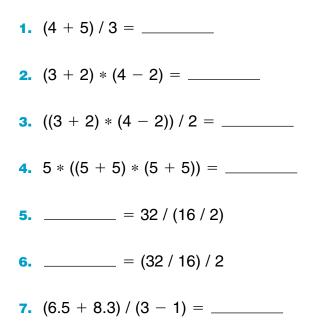
7 8

Date

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Review and Practice with Parentheses

Solve.



Make each sentence true by inserting parentheses.

8. 18 - 11 + 3 = 109. 18 - 11 + 3 = 410. 14 - 7 + 5 + 1 = 1311. 14 - 7 + 5 + 1 = 112. 14 - 7 + 5 + 1 = 113. 100 = 15 + 10 * 414. 4 = 24 / 4 + 215. 8 = 24 / 4 + 216. 10 - 4 / 2 * 3 = 2417. 10 - 4 / 2 * 3 = 1

Date	Time		
Math Boxes 7.6	MATH		
 a. A rectangle has a perimeter of 12 cm. One side is 4 cm long. Draw the rectangle. 	 2. a. Find an object in the room that is about 10 inches long. b. Find an object in the room that is about 10 centimeters long. 		
 What is the area of the rectangle you drew? 			
3. Find the radius and diameter of the circle.	 4. Use a calculator to rename each of the following in standard notation. a. 7³ =		
 5. Solve. Do not use a calculator. a. 243 b. 385 c. 1,006 + 477 + 948 - 597 			

500

Time

Math Message

The game *500* is a bat-and-ball game for two or more players. One player hits balls to the other players. The other players score points by catching the hit balls. Scoring is shown at the right.

Catch	Points
fly	100
one bounce	75
two bounces	50
grounder	25

If a player drops a ball, then the points are subtracted. For example, if a player tries to catch a fly ball and drops it, then 100 points are subtracted from the player's score.

The first player to reach 500 points becomes the next batter and the game starts over.

Sometimes, players have to go "in the hole." This happens when they miss a catch worth more points than they have. For example, if the first hit of the game is a fly and a player misses it, that player is 100 points "in the hole."

1. Complete the following table for a game of 500.

Action	Points Scored	Total Score
caught grounder	+25	25
missed fly	-100	75 in the hole
caught two-bouncer	+50	
caught fly		
missed fly		
missed one-bouncer		
missed fly		
caught fly		
caught fly		
caught fly		
missed one-bouncer		
caught fly		

2. Evan was 125 in the hole. How might he have gotten that score?_

Using Counters to Show an Account Balance

Use *Math Masters,* page 96. Shade the \pm squares with a regular pencil and the \equiv squares with a red pencil or crayon. Then cut out the squares.

- Each \pm counter represents \$1 of cash on hand.

Your **account balance** is the amount of money that you have or that you owe. If you have money in your account, your balance is "**in the black.**" If you owe money, your account is "**in the red.**"

1. Suppose you have this set of counters. + + + + + - - -

a. What is your account balance? _____

b. Are you "in the red" or "in the black"?

2. Use \pm and \Box counters to show an account with a balance of +\$5. Draw a picture of the counters below.

3. Use \pm and \Box counters to show an account with a balance of -\$8. Draw a picture of the counters below.

Use + and - counters to show an account with a balance of \$0. Draw a picture of the counters below.

Addition of Positive and Negative Numbers

Use your counters to help you solve these problems. Draw \pm and \Box counters to show how you solved each problem.

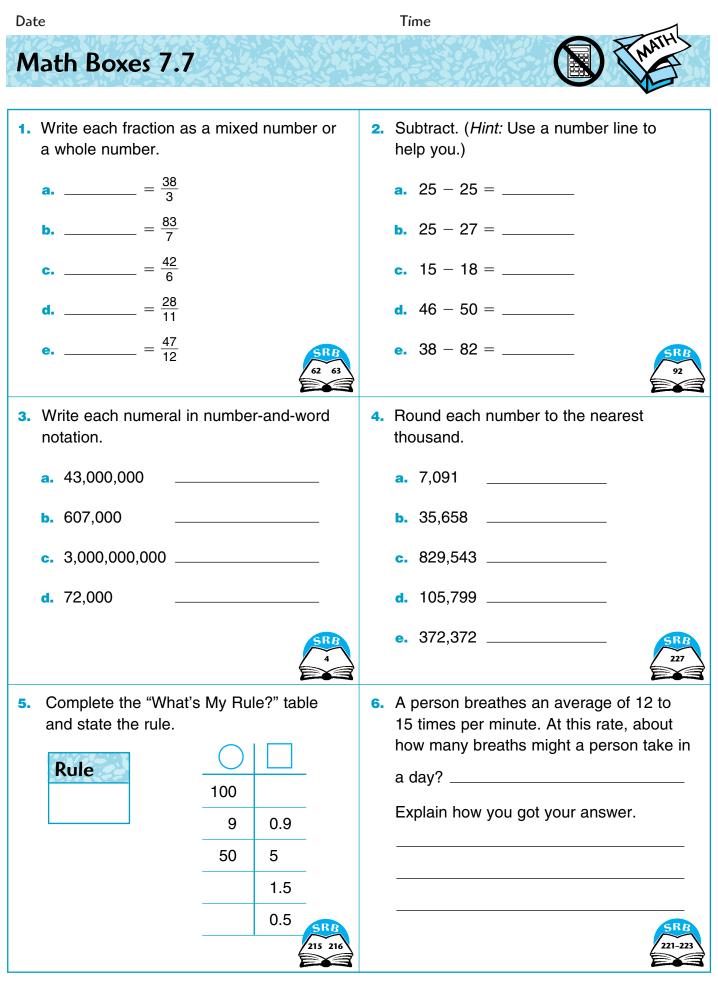
1. +8 + (-2) =	
2. -4 + (-5) =	
3. -3 + (+7) =	
Solve these addition problems.	
4. 50 + (-30) =	5. = -50 + 30
6. -16 + 10 =	7. = 16 + (-10)
8. -9 + (-20) =	9. = -15 + 15
10. 27 + (-18) =	11. = -43 + (-62)
12. -17 + (-17) =	13. = -55 + 32

Challenge

The temperature at sunset was 13°C. During the night, the temperature dropped 22°C. Write a number model and figure out the temperature at sunrise the next morning.

Number model:	

Answer: _____



Find	ing	Ral	and	-00	
	""g	Val	an	LES	

In the following problems, use your \$1 cash cards as \pm counters and your \$1 debt cards as \Box counters. The balance is the total value of the combined \pm and \Box counters.

Draw a picture of the \pm and \Box counters to show how you found each balance.

1. You have 3 \Box counters. Add 6 \pm counters.

Balance = _____ counters

2. You have 5 \pm counters. Add 7 \Box counters.

3. You have 5 \pm counters. Add 5 \equiv counters.

Balance = _____ counters

- **4.** Show a balance of -7 using 15 of your \pm and \Box counters.
- 5. You have 7 \Box counters. Take away 4 \Box counters.

6. You have 7 + counters. Take away 4 - counters.

Balance = _____ counters

7. You have 7 \Box counters. Take away 4 \pm counters.

Balance = _____ counters

2.

Date

1.

Adding and Subtracting Numbers

You and your partner combine your \boxdot and \boxdot counters. Use the counters to help you solve the problems.

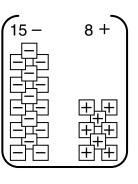
13 +	5 - `

Balance = $_$ If 4 \Box counters are subtracted from the container, what is the new balance?

New balance = _____

Number model: _____

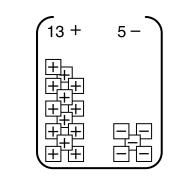




Balance = _____ If 3 + counters are subtracted from the container, what is the new balance?

New balance = _____

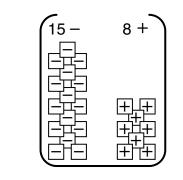
Number model: _____



Balance = _____ If 4 + counters are added to the container, what is the new balance?

New balance = _____

Number model: _____



Balance = $_$ If 3 \Box counters are added to the container, what is the new balance?

New balance = _____

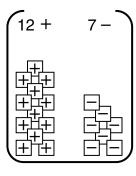
Number model: _____

6.

8.

Adding and Subtracting Numbers (cont.)

5.

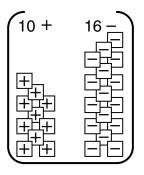


Balance = _____ If 6 \Box counters are subtracted from the container, what is the new balance?

New balance = _____

Number model: _____

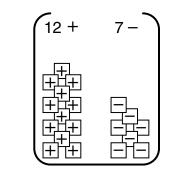
7.



Balance = _____ If 2 \Box counters are subtracted from the container, what is the new balance?

New balance = _____

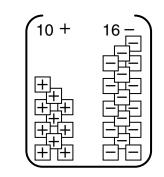
Number model: _____



Balance = _____ If 6 + counters are added to the container, what is the new balance?

New balance = _____

Number model: _____



Balance = _____ If 2 + counters are added to the container, what is the new balance?

New balance = _____

Number model: _____

9. Write a rule for subtracting positive and negative numbers.

Subtraction Problems

Rewrite each subtraction problem as an addition problem. Then solve it.

1.	100 - 45 =	/00 + (-45)	_ =
2.	-100 - 45 =		_ =
3.	160 - (-80) =		_ =
4.	9 - (-2) =		_ =
5.	-4 - (-2) =		_ =
6.	-15 - (-30) =		_ =
7.	8 - 10 =		_ =
8.	-20 - (-7) =		_ =
9.	$\pi - (-\pi) =$		_ =
10.	0 - (-6.1) =		_ =

Challenge

11. The Healthy Delights Candy Company specializes in candy that is wholesome and good for you. Unfortunately, they have been losing money for several years. During the year 2000, they lost \$12 million, ending the year with a total debt of \$23 million.

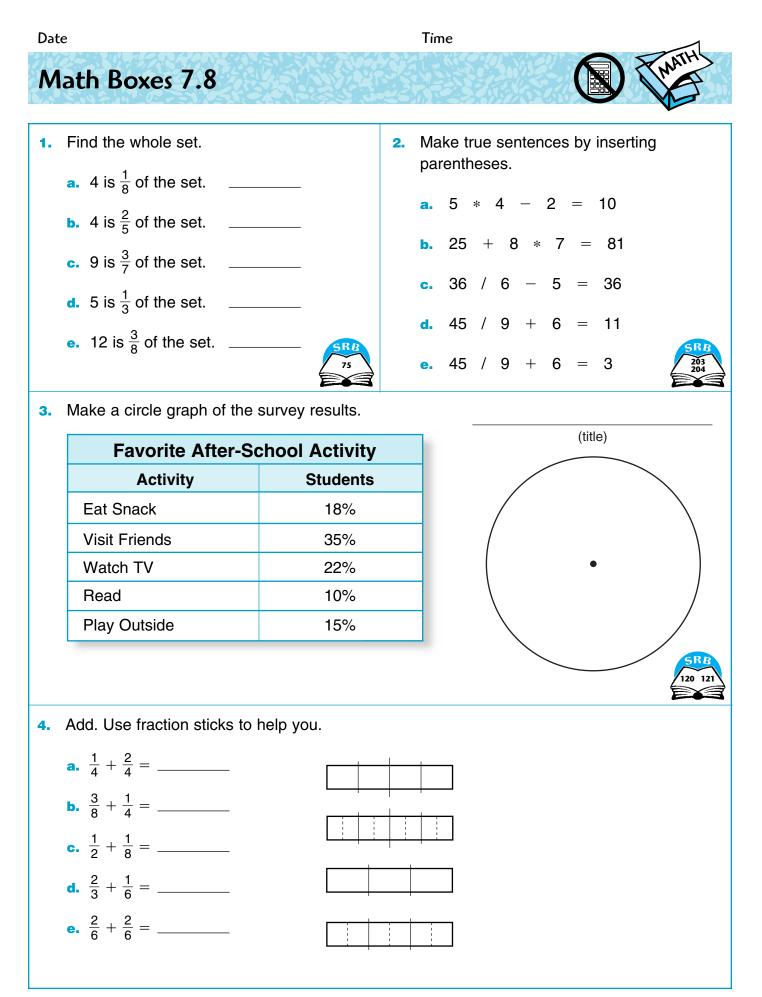
a. What was Healthy Delights' total debt at the beginning of 2000?

Write a number model that fits this problem.

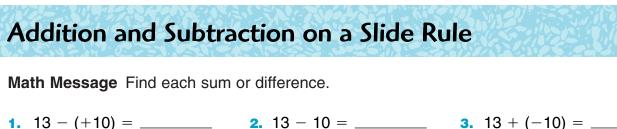
12. In 2001, Healthy Delights is expecting to lose \$8 million.

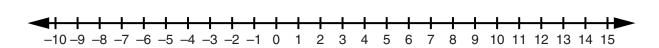
a. What will Healthy Delights' total debt be at the end of 2001?

Write a number model that fits this problem.



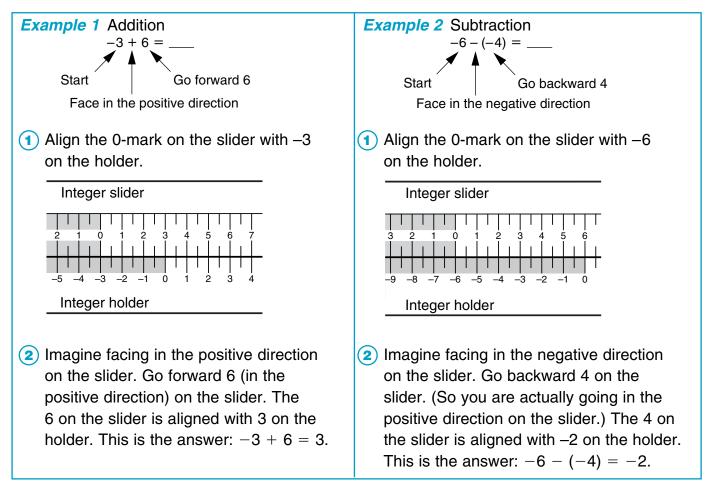
Use with Lesson 7.8.





Slide Rule Problems

Date



Use your slide rule to solve each problem.

4. 12 – 17 =	5. 12 + (-17) =	6. 10 - (-4) =
7. 10 + 4 =	8. -10 - (-5) =	9. 6 - 13 =
10. -2 + (-13) =	11. -5 - 10 =	12. -8 + 8 =
13. -8 - 8 =	14. -8 + (-8) =	15. -8 - (-8) =

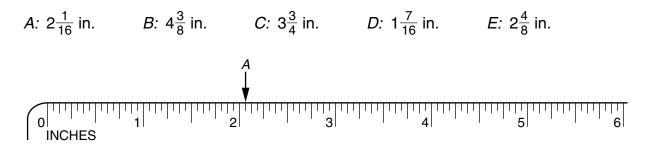
242

Date

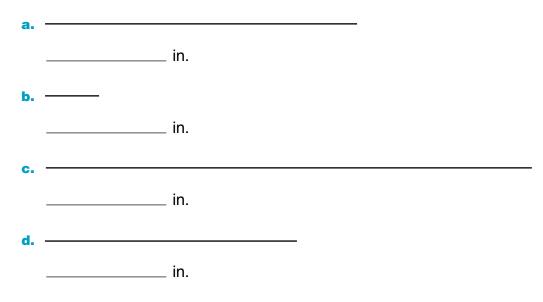
Time

Using a Ruler

1. Mark each of these lengths on the ruler shown below. Write the letter above your mark. Point *A* has been done for you.



2. Measure the following line segments to the nearest $\frac{1}{16}$ of an inch.



- **3.** Draw a line segment that is $4\frac{3}{16}$ inches long.
- 4. Draw a line segment that is $3\frac{1}{2}$ inches long.
- **5.** Complete these ruler puzzles.

Example
$$\frac{1}{4}$$
 in. $= \frac{x}{8}$ in. $= \frac{y}{16}$ in. $x = 2$ $y = 4$
a. $\frac{6}{8}$ in. $= \frac{x}{16}$ in. $= \frac{3}{y}$ in. $x = y = -$
b. $3\frac{2}{8}$ in. $= 3\frac{m}{4}$ in. $= 3\frac{4}{n}$ in. $m = n = -$
c. $\frac{6}{r}$ in. $= \frac{12}{s}$ in. $= \frac{t}{4}$ in. $r = s = t = -$

Use with Lesson 7.9.

Date	lime
Math Boxes 7.9	The the second s
1. Write each mixed number as a fraction. a	 2. Subtract. (<i>Hint:</i> Use a number line to help you). a. 32 - 38 = b. 14 - 21 = c. 84 - 85 = d. 36 - 52 = e. 40 - 73 = e. 40 - 73 = 4. Round each number to the nearest tenth. a. 45.06 b. 29.95 c. 1.005
 5. Complete the "What's My Rule?" table and state the rule. Rule 28 7 16 1 5 0 	 Marcus had \$5.00 to spend on lunch. He bought a hot dog for \$1.75 and some french fries for \$0.69. How much money did he have left for dessert?

Entering Negative Numbers on a Calculator

Math Message

2.	Repeat the steps in Problem 1 with other numbers.
3.	a. What is the opposite of 5? Enter the opposite of 5 in the calculator.
	 What is the opposite of the opposite of 5? Enter this number in the calculator, using the key.
4.	What does the key do?
١dd	ition and Subtraction Using a Calculator

Use your calculator to solve each problem. Record how you did it.

Example $12 + (-17) = -5$	Calculator Entry $/2 \oplus 0/7$ Enter
5. -10 - 17 =	
6. -10 + (-17) =	
7. -27 + 220 =	
8. 19 - 43 =	
9. -35 - (-35) =	
10. 72 + (-47) =	

Use with Lesson 7.10.

Addition and Subtraction Using a Calculator (cont.)

Solve. Use your calculator.

11. 3.65 – 2.02 =	12. 10 - (-5) =
13. -901 - 199 =	14. -7.1 + 18.6 =
15. -2 + (-13) + 7 =	16. 2 – 7 – (–15) =
17. 41 / 328 =	18. 3 * 3.14 =
19. -41 / 328 =	20. - (3 * 3.14) =
21. 41 * (7 + 2) =	22. 41 * (7 + (-2)) =

Number Stories

23. A salesperson is often assigned a quota. A quota is the dollar value of the goods that the salesperson is expected to sell.

Suppose a salesperson is \$3,500 below quota and then makes a sale of \$4,700.

Did the salesperson exceed or fall short of his or her quota?

Write a number model to figure out by how much the salesperson exceeded or fell short. (Use negative and positive numbers. Think about a number line with the quota at 0.)

Number model: _		

Solution: _____

24. Stock prices change every day. The first day, a stock's price went up $\frac{1}{4}$ dollar per share. The next day, it went down $\frac{1}{2}$ dollar. The third day, it went up $\frac{5}{8}$ dollar.

Did the value increase or decrease from the beginning of Day 1 to the end of

Day 3? _____

Write a number model to figure out by how much the stock increased or decreased over the 3-day period. (Use negative and positive numbers. Think about a number line with the Day 1 starting price at 0.)

Number model: _____

Solution: ___

Plotting Ordered Pairs

1. Plot the following ordered pairs on the grid below. As you plot each point, connect it with a line segment to the last one you plotted. (Use your ruler.)

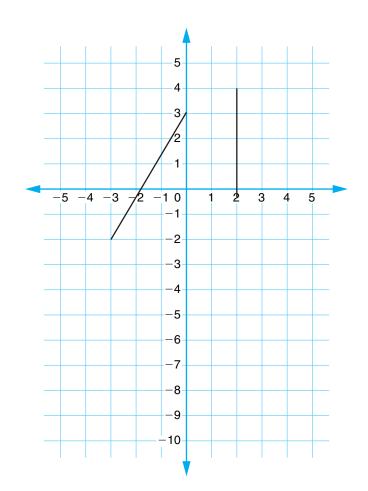
$$(0,3); (3,-2); (3,-8); (-3,-8); (-3,-2)$$

2. Plot the following ordered pairs on the grid below. As you plot each point, connect it with a line segment to the last one you plotted. (Use your ruler.)

 $(1,1\frac{1}{3}); (1,4); (2,4)$

3. Plot the following ordered pairs on the grid below. As you plot each point, connect it with a line segment to the last one you plotted. (Use your ruler.)

(1,-8); (1,-5); (-1,-5); (-1,-8)



Math Boxes 7.10

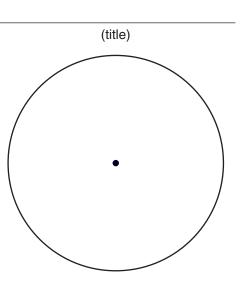
- There are 36 stamps in each package. How many stamps are there in ...
 - **a.** $\frac{3}{4}$ of a package? _____
 - **b.** $\frac{5}{6}$ of a package? _____
 - c. $\frac{2}{9}$ of a package? _____
 - **d.** $\frac{7}{12}$ of a package? _____
 - e. $\frac{2}{3}$ of a package? _____

 Make true sentences by inserting parentheses.

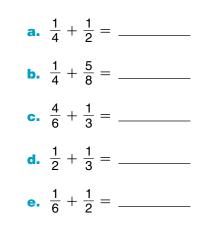
Time

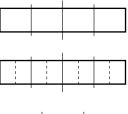
- **a.** 19 + 41 * 3 = 180
- **b.** 5 = 16 / 2 + 2 5
- **c.** -1 = 16 / 2 + 2 5
- **d.** $24 \div 8 + 4 * 3 = 6$
- **e.** $24 \div 8 + 4 * 3 = 15$
- 3. Make a circle graph of the survey results.

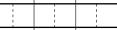
Time Spent on Homework			
Time	Percent of Students		
0–29 minutes	25		
30–59 minutes	48		
60–89 minutes	10		
90–119 minutes	12		
2 hours or more	5		



4. Add. Use fraction sticks to help you.







T	me to Reflect
1.	Look through your journal pages in this unit. Which pages do you think show your best work? Explain.
2.	Explain why you think we have negative numbers in our number system. Give
	examples to support your claims.
3.	List some of the skills and concepts you learned in this unit that you think are important to remember because you will use them in the future. Explain your answers.

Date	Time
Math Boxes 7.11	Thrather the
1. Rewrite each fraction pair with common denominators.a. $\frac{2}{5}$ and $\frac{3}{7}$ b. $\frac{4}{12}$ and $\frac{6}{9}$ c. $\frac{8}{10}$ and $\frac{10}{15}$	 2. a. What is the perimeter of the rectangle?
 a. 26 b. 71 <u>* 32</u> <u>* 58</u> 	c. 93 <u>* 47</u>
4. Write each fraction as a mixed number or a whole number. a. $\frac{39}{4} = $ b. $\frac{62}{7} = $ c. $\frac{45}{6} = $ d. $\frac{200}{5} = $ e. $\frac{83}{9} = $	5. Find the whole set. a. 10 is $\frac{1}{5}$ of the set. b. 12 is $\frac{3}{4}$ of the set. c. 8 is $\frac{2}{7}$ of the set. d. 15 is $\frac{5}{8}$ of the set. e. 9 is $\frac{3}{5}$ of the set.

Comp	aring	Erac	tione
Comp	anng	1 I au	LIUID
	U		

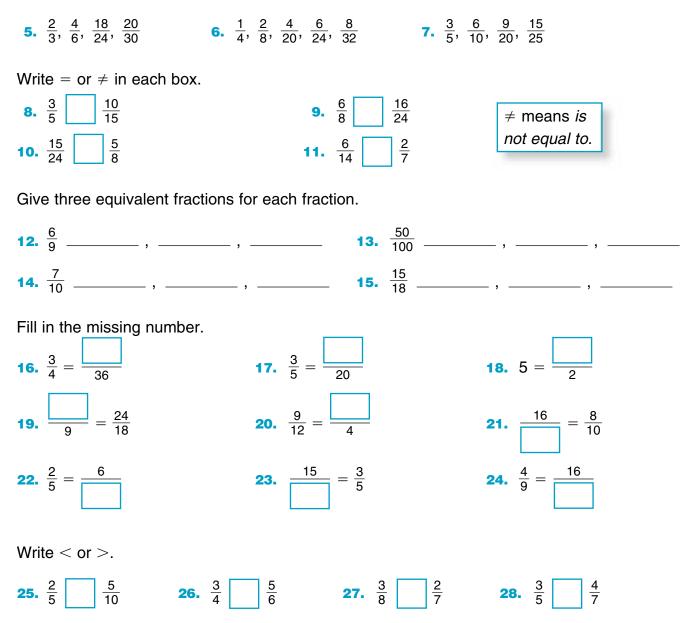
Math Message

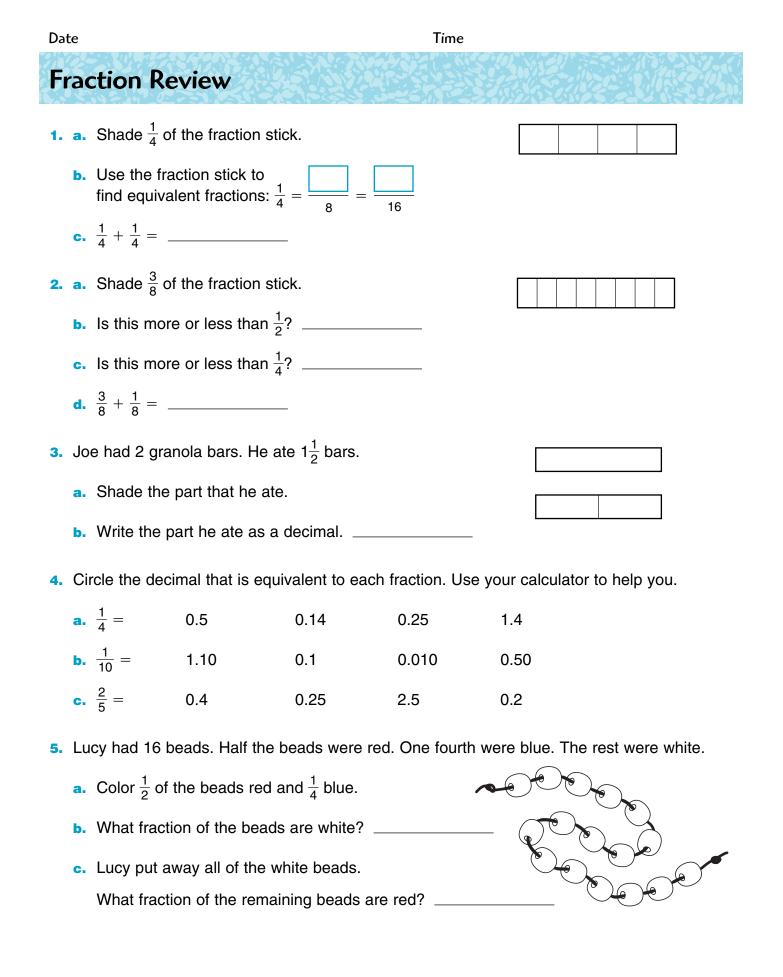
Write < or >. Be prepared to explain how you decided on each answer.

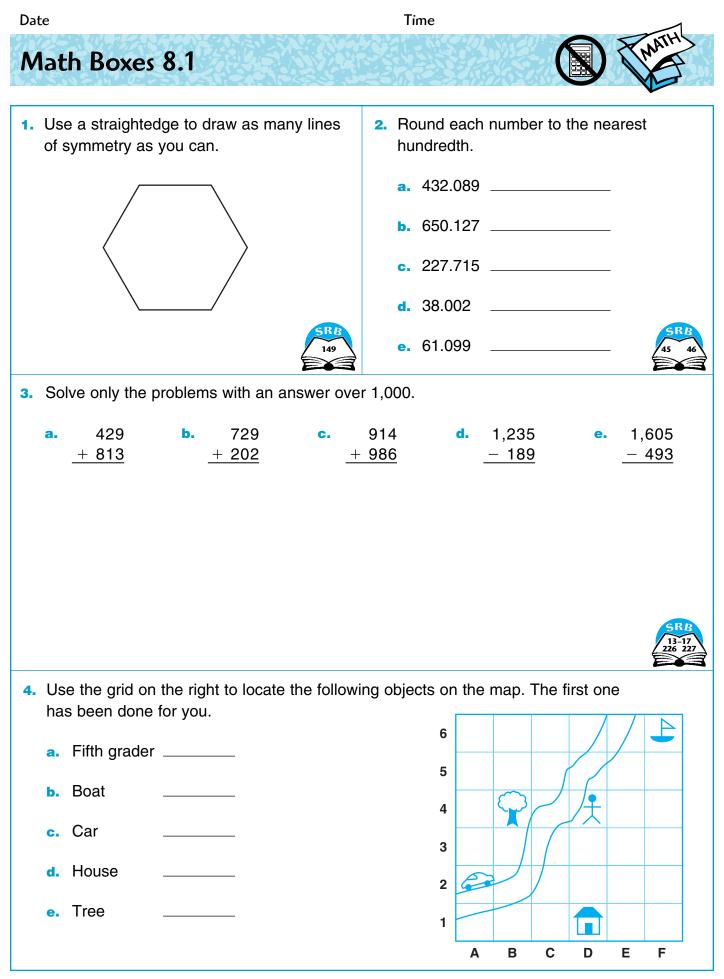


Equivalent Fractions

Cross out the fraction in each list that is not equivalent to the other fractions.





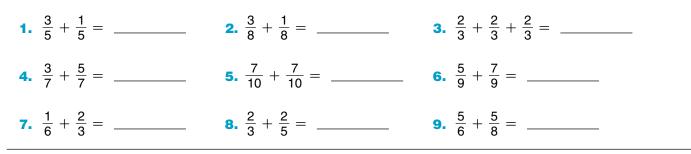


Use with Lesson 8.1.

Addition of Fractions

Math Message

Add. Write the sums in simplest form.

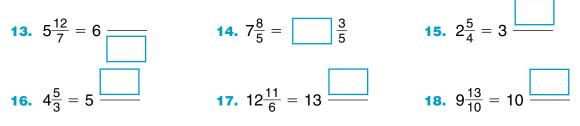


Addition of Mixed Numbers

Add. Write each sum as a whole number or mixed number.

10. $1\frac{3}{5}$	11. $1\frac{1}{2}$	12. $2\frac{1}{4}$
$+ 1\frac{1}{5}$	$+\frac{1}{2}$	$+ 3\frac{3}{4}$

Fill in the missing numbers.



Add. Write each sum as a mixed number in simplest form.

19. $3\frac{2}{3}$	20. 4 $\frac{6}{7}$	21. 3 $\frac{4}{9}$
$+ 5\frac{2}{3}$	$+ 2\frac{4}{7}$	$+ 6\frac{8}{9}$

Addition of Mixed Numbers (cont.)

To add mixed numbers in which the fractions do not have the same denominator, you must first rename one or both fractions so that both fractions have a common denominator.

Example $2\frac{3}{5} + 4\frac{2}{3} = ?$

- Find a common denominator: The QCD of $\frac{3}{5}$ and $\frac{2}{3}$ is 5 * 3 = 15.
- Write the problem in vertical form and rename the fractions:

$$\begin{array}{c} 2\frac{3}{5} \\ + 4\frac{2}{3} \\ \end{array} \xrightarrow{} \begin{array}{c} 2\frac{9}{15} \\ + 4\frac{10}{15} \\ \end{array} \\ \hline \begin{array}{c} 6\frac{19}{15} \\ \end{array} \end{array}$$

- Add.
- Rename the sum. $6\frac{19}{15} = 7\frac{4}{15}$

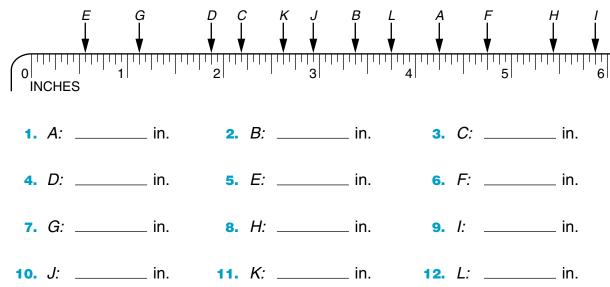
Add. Write each sum as a mixed number in simplest form. Show your work.

1.
$$2\frac{1}{3} + 3\frac{1}{4} =$$

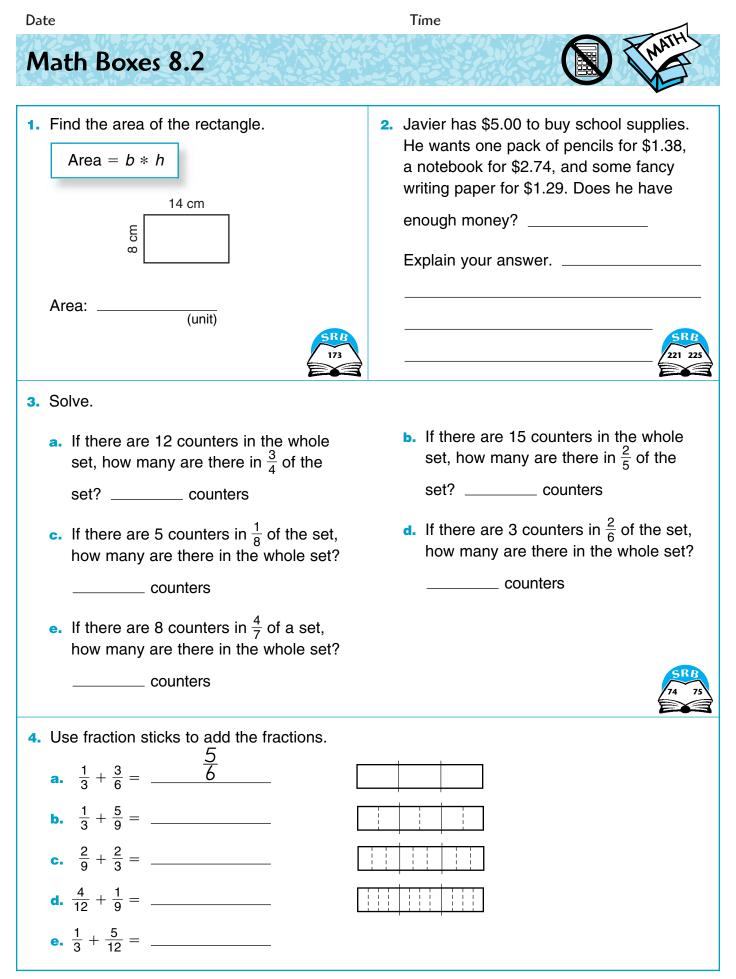
2. $5\frac{1}{2} + 2\frac{2}{5} =$
3. $6\frac{1}{3} + 2\frac{4}{9} =$
4. $1\frac{1}{2} + 4\frac{3}{4} =$
5. $7\frac{1}{4} + 2\frac{5}{6} =$
6. $3\frac{5}{6} + 3\frac{3}{4} =$

Reading a Ruler

On the ruler below, points A through L mark distances from the beginning of the ruler (0 inches). Give the distance from 0 for each point. Point A has been done for you.



Pick four of the points in Problems 1-12. For each point, write an equivalent name for its distance from 0.



Subtraction of Mixed Numbers

Math Message

Subtract.

1. $3\frac{3}{4}$	2. $4\frac{4}{5}$	3. 7 $\frac{5}{6}$
$-1\frac{1}{4}$		$-2\frac{2}{6}$

Renaming and Subtracting Mixed Numbers

Fill in the missing numbers.

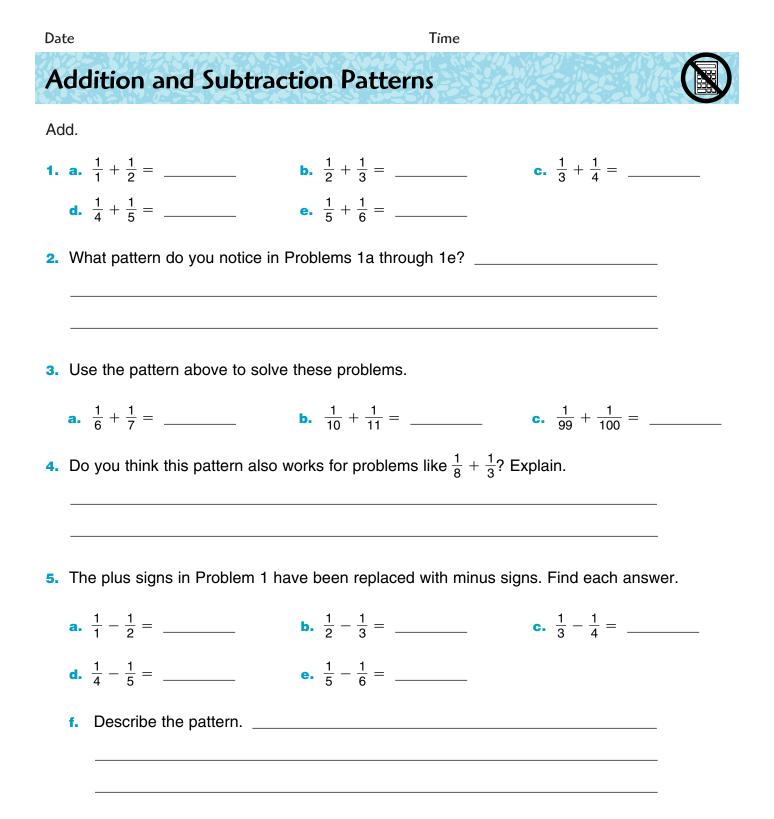


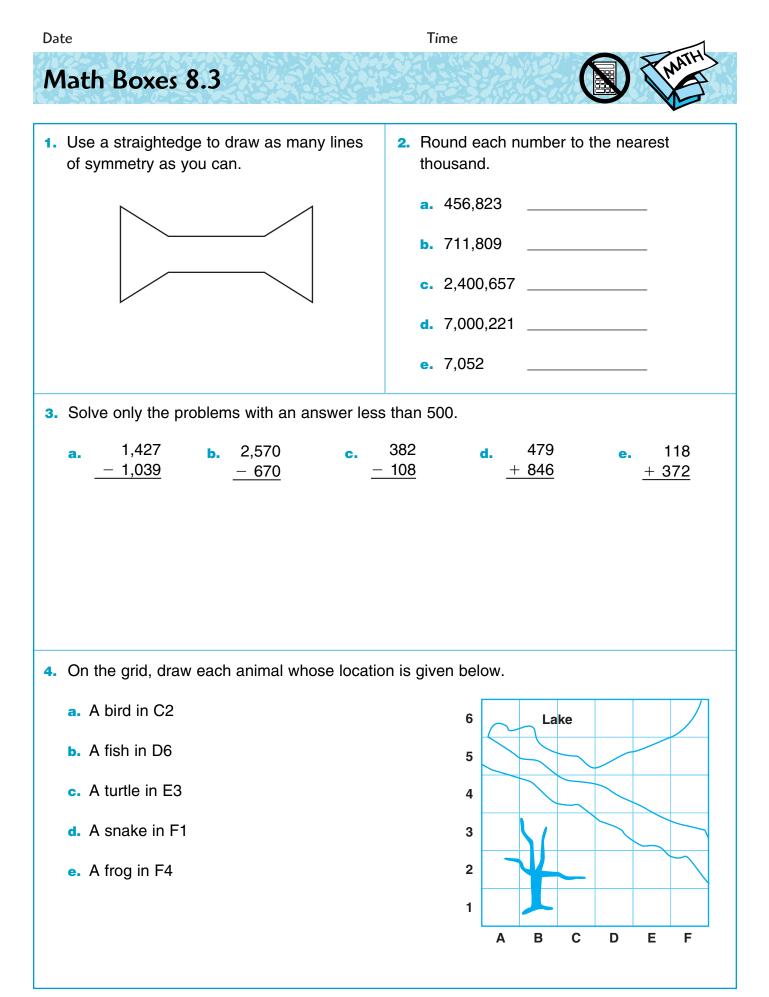
Subtract. Write your answers in simplest form. Show your work.

8.
$$8 - \frac{1}{3} =$$
 9. $5 - 2\frac{3}{5} =$

10.
$$7\frac{1}{4} - 3\frac{3}{4} =$$
 11. $4\frac{5}{8} - 3\frac{7}{8} =$

12.
$$6\frac{2}{9} - 4\frac{5}{9} =$$
 13. $10\frac{3}{10} - 5\frac{7}{10} =$ **.....**





Calculator Key Investigation

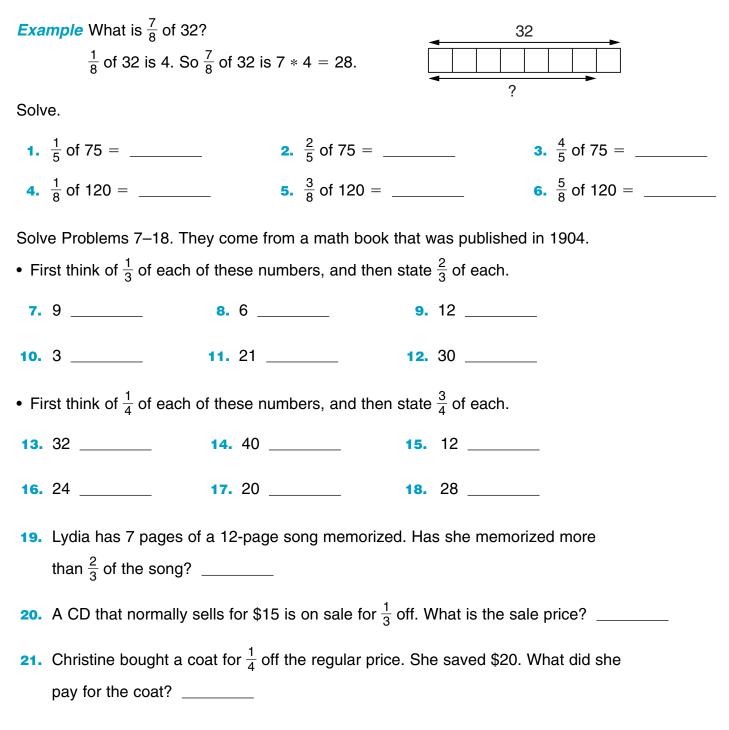
Explore the seven function keys on your calculator. Use the sample keystrokes to help you find what each key does.

Key(s)	Sample Keystrokes	Function of Key(s)
<u>Unit</u>)	Unit n a Enter	
<u>।</u> and ত্র	2 🛄 3 🖪 🖽	
F↔D	4 <u>n</u> 5 <u>a</u> ^{Enter} F⊷D F↔D	
Simp	4 <u>n</u> 12 <u>a</u> Simp ^{Enter} Simp ^{Enter}	
(Uª+ª)	14 <u>n</u> 3 <u>d</u> ^{Enter} Simp ^{Enter} Uaa≁a Uaa≁a	
Fac	4 <u>n</u> 8 Simp ^{Enter} (Fac	

Use with Lesson 8.4.

Finding a Fraction of a Number

One way to find a fraction of a number is to use a **unit fraction**. (A unit fraction is a fraction with 1 in the numerator.) You can also use a diagram to help you understand the problem.



Date	Time
Math Boxes 8.4	D Thatte
1. Find the area of the rectangle. $Area = b * h$ $12 m$ E_{O} Area:(unit)	 2. Julie makes \$4.00 per week for doing the dishes every night. She paid her sister Amy \$0.75 each time Amy did the dishes for her. Is that a fair price? Explain your answer.
 3. Solve. a. If there are 18 counters in the whole set, how many are there in ⁵/₆ of the set? counters c. If there are 6 counters in ²/₇ of the set, how many are there in the whole set? counters e. If there are 9 counters in ³/₈ of the set, how many are there in the whole set? counters 	 b. If there are 21 counters in the whole set, how many are there in ²/₃ of the set? counters d. If there are 10 counters in ¹/₅ of the set, how many are there in the whole set? counters
4. Use fraction sticks to add the fractions. a. $\frac{1}{4} + \frac{3}{3} = $ b. $\frac{1}{8} + \frac{1}{2} = $ c. $\frac{5}{8} + \frac{1}{4} = $ d. $\frac{5}{12} + \frac{1}{4} = $ e. $\frac{3}{4} + \frac{1}{12} = $	

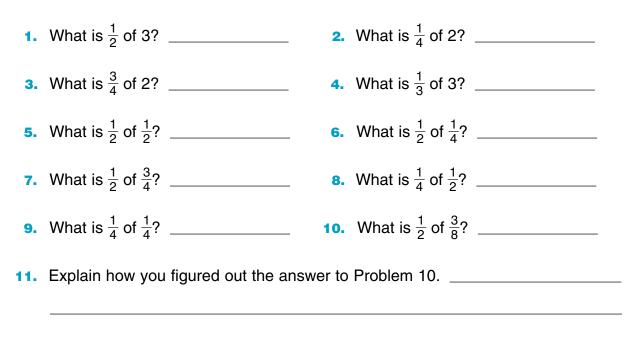
Use with Lesson 8.4.

Date		Time			
Math Boxes 8.	5			MATTH	>
1028 STA 224 122 2307	<u> </u>			M	
1. Name 2 objects that	are shaped like a	2. Divide mentally.			
	rectangular prism.				
		b. 795/5 $ ightarrow$			
		c. 496 / 4 $ ightarrow$			
		d. $283 \div 6 \rightarrow$			
		e. 1,625 ÷ 8 →			
3. Amanda found a car	n containing 237 domin	oes. A full set has 28 domin	oes. What	is the	
greatest number of o	complete sets that can l	be in the can?		(uni	
				(-,
Explain how you fou	nd your answer				_
					_
					_
				CRI	-
				21-24 224	
4. Complete the table.		5. Complete the "What's	My Rule?	" table ar	nd
Standard	Exponential	state the rule.	in	out	
Notation 10,000	Notation	Rule:	3	Jul	
	10 ³		8	40	
	10 ⁸		<u>1</u> 2		
1,000,000,000				50	
	10 ⁵		4		
	SRB 5	3		SR/ 215 2	216
				EX	3





Use the number line above to help you answer Problems 1–10.



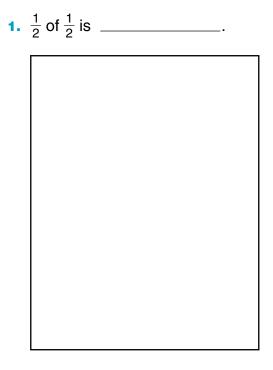


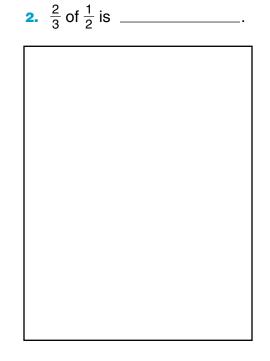
All (Winged) Creatures: Great and Small

The smallest bird is the bee hummingbird. It weighs about 2 grams (0.07 ounces) and is about 5.5 centimeters (2.2 inches) long. The largest bird is the ostrich. An adult ostrich can weigh more than 150 kilograms (330 pounds) and stand 2.5 meters (8 feet) tall. (It's no wonder that ostriches can't fly!) It would take more than 75,000 bee hummingbirds to balance one ostrich (if you could find a balance big enough). Source: Britannica Online

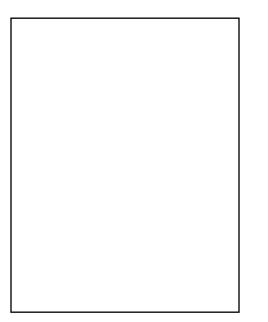
Paper-Folding Problems

Record your work for the four fraction problems you solved by paper folding. Sketch the folds and shading. Write an X on the parts that show the answer.

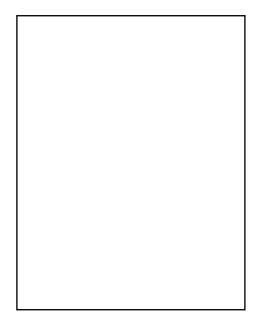




3. $\frac{1}{4}$ of $\frac{2}{3}$ is _____.

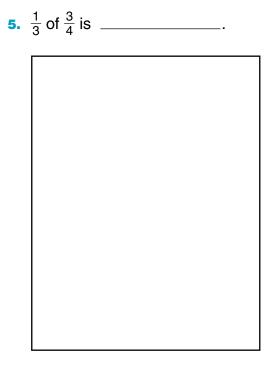


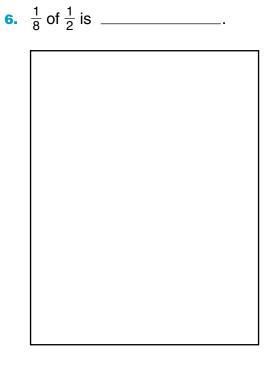
4. $\frac{3}{4}$ of $\frac{1}{2}$ is _____.



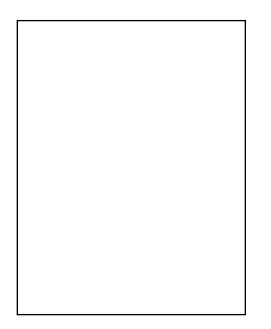
Paper-Folding Problems (cont.)

Solve these problems by paper folding. Sketch the folds and shading. Write an X on the parts that show the answer.

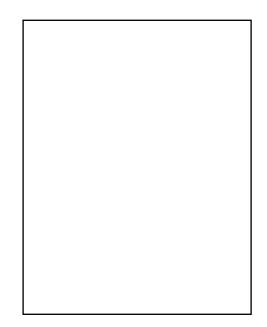


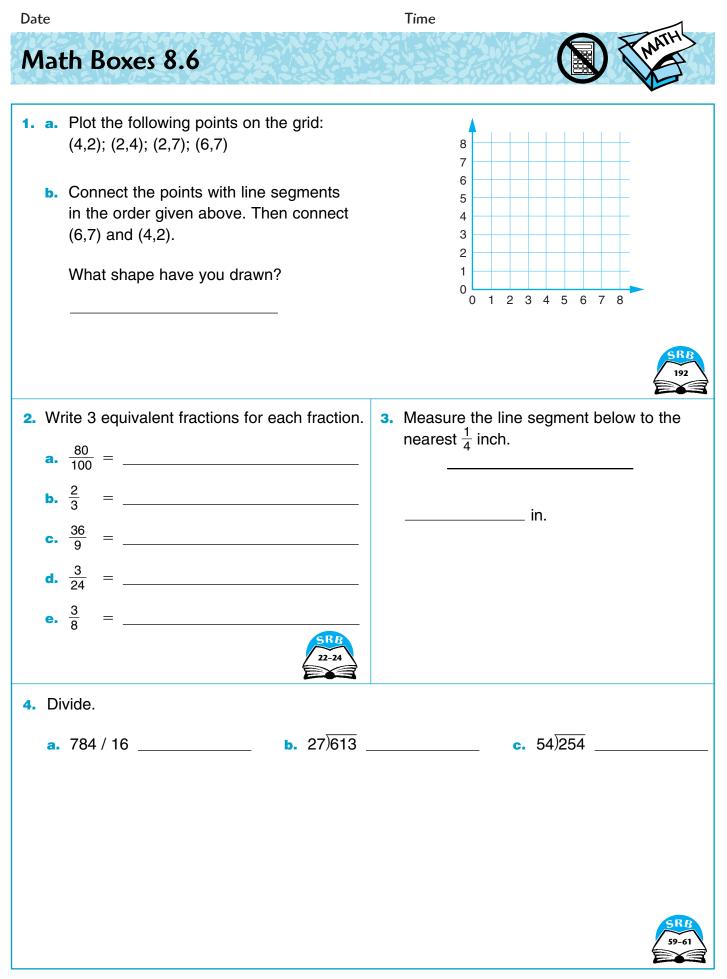


7. $\frac{5}{8}$ of $\frac{1}{2}$ is _____.



8. $\frac{3}{4}$ of $\frac{3}{4}$ is _____.



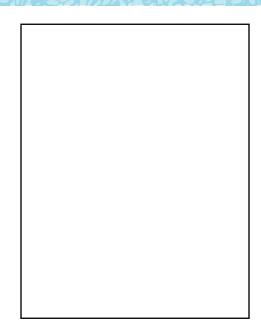


Use with Lesson 8.6.

Fraction Multiplication

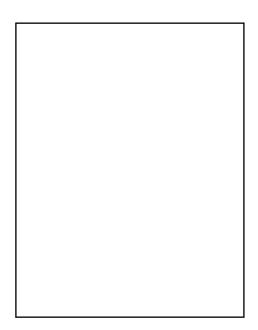
1. Use the rectangle at the right to sketch how you would fold paper to help you find $\frac{1}{3}$ of $\frac{2}{3}$.

What is $\frac{1}{3}$ of $\frac{2}{3}$? _____



2. Use the rectangle at the right to sketch how you would fold paper to help you find $\frac{1}{4}$ of $\frac{3}{5}$.

What is $\frac{1}{4}$ of $\frac{3}{5}$? ______



3. Rewrite $\frac{2}{3}$ of $\frac{3}{4}$ using the multiplication symbol *.

4. Rewrite the following using the multiplication symbol *.

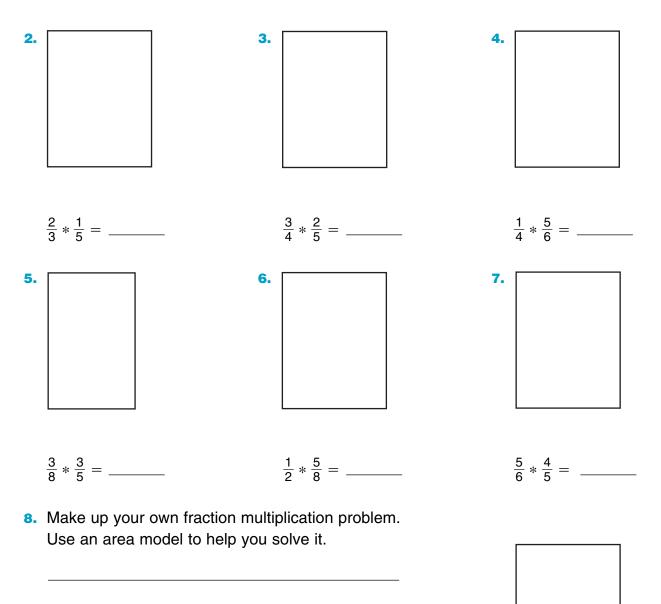


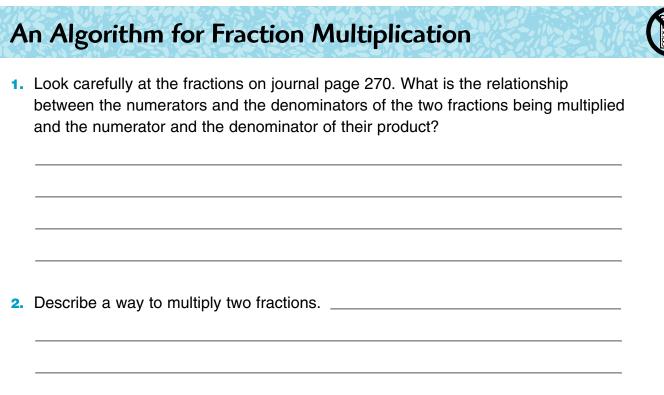
An Area Model for Fraction Multiplication

- **1.** Use the rectangle at the right to find $\frac{2}{3} * \frac{3}{4}$.
 - $\frac{2}{3} * \frac{3}{4} =$ _____

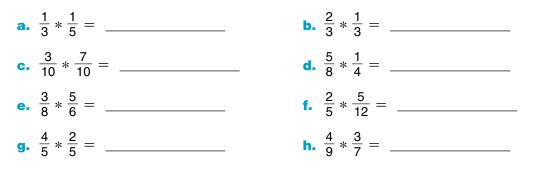
Your completed drawing in Problem 1 is called an area model.

Use area models to complete the following.





3. Multiply the following fractions, using the shortcut discussed in class.





Cashing in on Fractions

Torn money might be worth something. If you have $\frac{3}{5}$ or more of a bill, it can be redeemed for its full face value. If you have less than $\frac{3}{5}$ but more than $\frac{2}{5}$ of a bill, it can be redeemed for $\frac{1}{2}$ of its face value. If you have $\frac{2}{5}$ or less, your bill is worthless. Source: You Can't Count a Billion Dollars

Place-Value Practice

Find the missing number.

Date

- 1. The digit in the tens place is twice as big as the digit in the tenths place. The digit in the ones place is $\frac{1}{2}$ the digit in the tenths place. The digit in the hundredths place is $\frac{1}{2}$ the digit in the ones place. The digit in the hundreds place is the largest odd digit.
- 2. The digit in the hundreds place is a square number and it is odd. The digit in the tens place is 1 more than the square root of 16. The digit in the hundredths place is 0.1 larger than $\frac{1}{10}$ of the digit in the hundreds place. The digit in the theusendthe place is equivalent to $\frac{30}{10}$

The digit in the thousandths place is equivalent to $\frac{30}{5}$. The other digits are all 2s.

3. Record the calculator keystrokes you enter to make the changes described below.

Beginning Number	Change to	Keystrokes
12,204	15,204	12204
807,995	808,005	807995
2.112	2.712	2.112
17.054	18.104	17.054
34.921	35.021	34.921

Date

A Blast from the Past

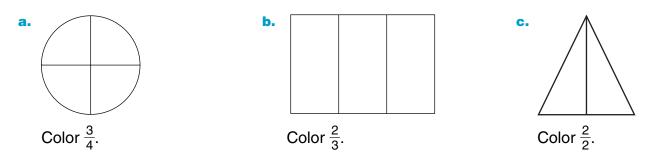
1. From Kindergarten Everyday Mathematics:

This slice of pizza is what fraction of the whole pizza?

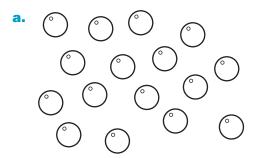


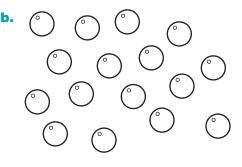
2. From First Grade Everyday Mathematics:

Write a fraction in each part of the diagrams below. Then color the figures as directed.



3. From Second Grade Everyday Mathematics:





Color $\frac{1}{4}$ of the beads.

Color $\frac{1}{8}$ of the beads.

- 4. From Third Grade Everyday Mathematics:
 - **a.** $\frac{1}{2}$ of $\frac{1}{4} =$ _____ **b.** $\frac{1}{8}$ of $\frac{1}{2} =$ _____ **c.** $\frac{1}{2}$ of $\frac{1}{8} =$ _____
- 5. From Fourth Grade Everyday Mathematics:
 - Cross out $\frac{5}{6}$ of the dimes.



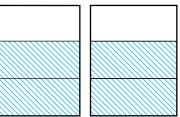
Date

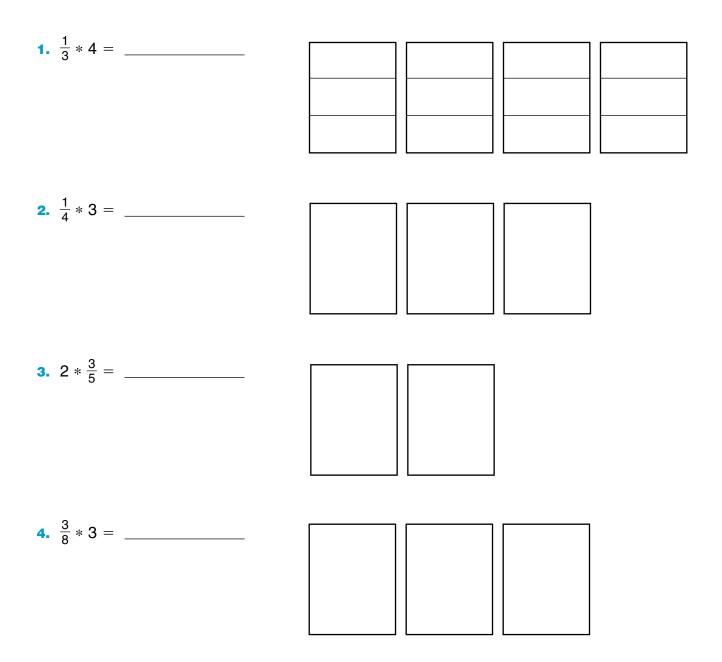
Time

Area Models

Draw an area model for each product. Then write the product as a fraction or as a mixed number.

Example $\frac{2}{3} * 2 = \frac{\frac{4}{3}}{\frac{1}{3}}$ or $1 \frac{\frac{1}{3}}{\frac{1}{3}}$





Using the Fraction Multiplication Algorithm

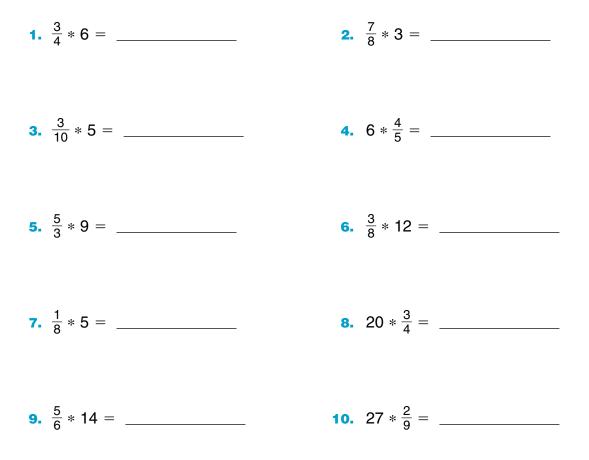
An Algorithm for Fraction Multiplication

$$\frac{a}{b} * \frac{c}{d} = \frac{a * c}{b * d}$$

The denominator of the product is the product of the denominators, and the numerator of the product is the product of the numerators.

Example $\frac{2}{3} * 2$ $\frac{2}{3} * 2 = \frac{2}{3} * \frac{2}{1}$ Think of 2 as $\frac{2}{1}$. $= \frac{2 * 2}{3 * 1}$ Apply the algorithm. $= \frac{4}{3}$, or $1\frac{1}{3}$ Calculate the numerator and denominator.

Use the fraction multiplication algorithm to calculate the following products.



Date		Time			
N	lath Boxes 8.7	7		Š	MATH
1.		•	2. Divide mentally. a. $472 \div 5 \rightarrow$ b. $384 / 6 \rightarrow$ c. $729 / 8 \rightarrow$ d. $543 \div 4 \rightarrow$ e. $576 \div 9 \rightarrow$		
3.	 Raphael bought 14 pounds of meat to make He made 5 hamburgers from each pound. E packages of buns did Raphael need? Explain your answer 		Buns come in packages of 8. I	How mar	וץ (unit)
4.	Complete the table.		 Complete the "What's M state the rule. 	ly Rule?"	' table and
	Standard Notation	Exponential Notation	Rule:	in	out
		10 ⁵		48	
		10 ⁹		40	5
	1,000,000	10			<u>1</u> 8
	10,000				0
	,	10 ⁷		16	

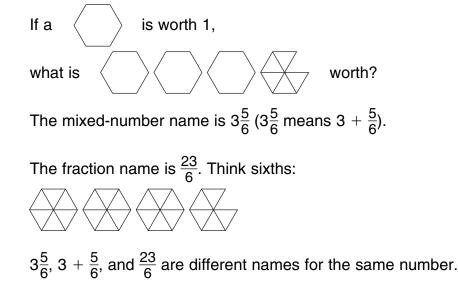
Date

Review Converting Fractions to Mixed Numbers

Time

You know that fractions larger than 1 can be written in several ways.

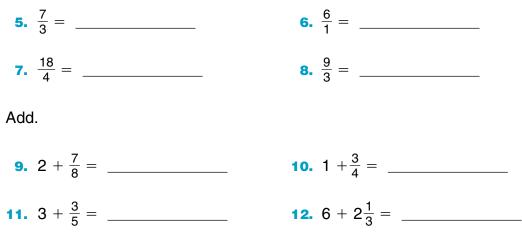
Example



Write the following mixed numbers as fractions.



Write the following fractions as mixed or whole numbers.





Multiplication of Fractions and Mixed Numbers

Examples Using Partial Products			
$3\frac{1}{4}*\frac{2}{5}=(3+\frac{1}{4})*\frac{2}{5}$			
$3 * \frac{2}{5} = \frac{6}{5} = 1 \frac{1}{5}$			
$\frac{1}{4} * \frac{2}{5} = \frac{2}{20} = +\frac{1}{10}$			
$1\frac{3}{10}$			

Examples Converting Mixed Numbers to Fractions		
$2\frac{1}{3} * 2\frac{1}{2} = \frac{7}{3} * \frac{5}{2}$	$3\frac{1}{4} * \frac{2}{5} = \frac{13}{4} * \frac{2}{5}$	
$=\frac{35}{6}=5\frac{5}{6}$	$=\frac{26}{20}=1\frac{6}{20}=1\frac{3}{10}$	

Solve the following fraction and mixed-number multiplication problems.

 $3\frac{5}{6}$ "

1.
$$3\frac{1}{2} * 2\frac{1}{5} =$$

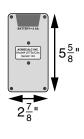
The surface of a calculator is approximately a rectangular prism.
 The back face has an area of about

5. The area of a

about

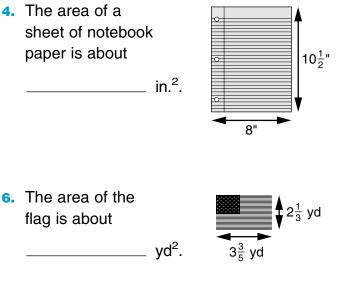
computer disk is

_____ in.².



 $3\frac{1}{2}$

2. $10\frac{3}{4} * \frac{1}{2} =$ _____



7. Is the area of the flag greater or less than the area of your desk or tabletop?

in.2

Date

Time

Track Records on the Moon and the Planets

Every moon and planet in our solar system pulls objects toward it with the force called **gravity.**

In a recent Olympic games, the winning high jump was 7 feet 8 inches, or $7\frac{2}{3}$ feet. The winning pole vault was 19 feet. Suppose that the Olympics were held on Earth's Moon, or on Jupiter, Mars, or Venus. What height might we expect for a winning high jump or a winning pole vault?

 On the Moon, one could jump about 6 times as high as on Earth. What would be the height of the winning ...

	high jump? About	feet	pole vault? Ab	bout	feet
2.	On Jupiter, one could jump about What would be the height of the w	-	as on Earth.		
	high jump? About	feet	pole vault? Ab	bout	feet
3.	On Mars, one could jump about 2 What would be the height of the w	0	s high as on Ea	arth.	
	high jump? About	feet	pole vault? At	bout	feet
4.	On Venus, one could jump about What would be the height of the w	1	as high as on E	Earth.	
	high jump? About	_ feet	pole vault? Ab	bout	feet
5.	Is Jupiter's pull of gravity stronger Explain your reasoning.	or weake	er than Earth's?	?	
-9					

Challenge

6. The winning pole-vault height given above was rounded to the nearest whole number. The actual winning height was 19 feet $\frac{1}{4}$ inch. If you used this actual measurement, about how high would the winning jump be ...

on the Moon? _____ on Jupiter? _____

on Mars? _____ on Venus? _____

Use with Lesson 8.8.



—

Date	lime
Math Boxes 8.8	The second
 a. Plot the following points on the grid: (-3,-3); (1,1); (4,1); (0,-3) b. Connect the points with line segments in the order given above. Then connect (-3,-3) and (0,-3). What shape have you drawn? 	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
2. Write 3 equivalent fractions for each fraction. a. $\frac{2}{5} =$	 Measure line segment <i>IT</i> below to the nearest tenth of a centimeter. I T
 4. Divide. a. 12)597 b. 22)3,840 _ 	c. 15)1,630

Finding a Percent of a Number

 The Madison Middle School boys' basketball team has played 5 games. The table at the right shows the number of shots taken by each player and the percent of shots that were baskets. Study the example. Then calculate the number of baskets made by each player.

Example

Bill took 15 shots. He made a basket on 40% of these shots.

40% = $\frac{40}{100}$, or $\frac{4}{10}$ $\frac{4}{10}$ of 15 = $\frac{4}{10} * \frac{15}{1} = \frac{4 * 15}{10 * 1} = \frac{60}{10} = 6$

Bill made 6 baskets.

 On the basis of shooting ability, which five players might you select as the starting lineup for the next basketball game?

Player	Shots Taken	Percent Made	Baskets
Bill	15	40%	6
Amit	40	30%	
Josh	25	60%	
Kevin	8	75%	
Mike	60	25%	
Zheng	44	25%	
André	50	10%	
David	25	20%	
Bob	18	50%	
Lars	15	20%	
Justin	28	25%	

- 3. What other factors might you consider when making this decision?
- 4. Which player(s) might you encourage to shoot more often? _____

Why? _____

Which player(s) might you encourage to pass more often?

Why? _____

Calculating a Discount

Example The list price for a toaster is \$45. The toaster is sold at a 12% discount (12% off the list price). What are the savings? (*Reminder:* $12\% = \frac{12}{100} = 0.12$)

Paper and pencil:

12% of \$45 $= \frac{12}{100} * 45 = \frac{12}{100} * \frac{45}{1}$ $= \frac{12 * 45}{100 * 1} = \frac{540}{100}$ = \$5.40

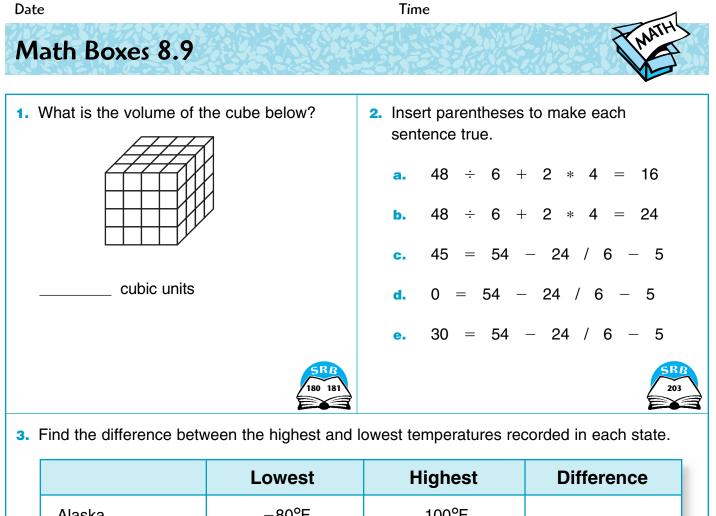
Calculator:

Enter 0.12 \otimes 45 Enter; interpret answer 5.4 as \$5.40.

First use your percent sense to estimate the discount for each item in the table below. The **discount** is the amount by which the list price of an item is reduced. It is the amount the customer saves.

Then use your calculator or paper and pencil to calculate the discount. (If necessary, round to the nearest cent.)

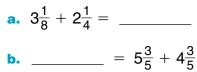
Item	List Price	Percent of Discount	Estimated Discount	Calculated Discount
Clock radio	\$33.00	20%		
Calculator	\$60.00	7%		
Sweater	\$20.00	42%		
Tent	\$180.00	30%		
Bicycle	\$200.00	17%		
Computer	\$980.00	25%		
Skis	\$325.00	18%		
Double CD	\$29.99	15%		
Jacket	\$110.00	55%		



Alaska	-80°F	100°F	
Arizona	-40°F	127°F	
Nebraska	-47°F	118°F	
South Dakota	-58°F	120°F	

Source: World Almanac

4. Add. Do not use a calculator.



- **c.** _____ = $1\frac{7}{8} + 2\frac{1}{2}$
- **d.** _____ = $\frac{8}{10} + 3\frac{5}{4}$ **e.** _____ = $\frac{7}{8} + \frac{1}{5}$

5. List the factors of 142.

Unit Fractions and Unit Percents

1.	If 12 counters are $\frac{1}{5}$ of a set, how many counters are in the set?	 counters
2.	If 15 counters are $\frac{1}{7}$ of a set, how many counters are in the set?	 counters
3.	If 31 pages are $\frac{1}{8}$ of a book, how many pages are in the book?	 pages
4.	If 13 marbles are 1% of the marbles in a jar, how many marbles are in the jar?	 marbles
5.	If \$5.43 is 1% of the cost of a TV, what does the TV cost?	 dollars
6.	If 84 counters are 10% of a set, how many counters are in the set?	 counters
7.	After 80 minutes, Dorothy had read 120 pages of a 300-page book. If she continues reading at the same rate, about how long will it take her to read the entire book?	 minutes
8.	Eighty-four people attended the school concert. This was 70% of the number expected to attend. How many people were expected to attend?	 people

Challenge

In its most recent game, the Lincoln Junior High basketball team made 36 baskets, which was 48% of the shots team members tried.
How many shots did they try?

_____ shots

Using a Unit Fraction or a Unit Percent to Find the Whole

1. Six jars are filled with cookies. The number of cookies in each jar is not known. For each clue given below, find the number of cookies in the jar.

Clue	Number of Cookies in Jar
a. $\frac{1}{2}$ jar contains 31 cookies.	
b. $\frac{2}{8}$ jar contains 10 cookies.	
c. $\frac{3}{5}$ jar contains 36 cookies.	
d. $\frac{3}{8}$ jar contains 21 cookies.	
e. $\frac{4}{7}$ jar contains 64 cookies.	
f. $\frac{3}{11}$ jar contains 45 cookies.	

2. Use your percent sense to estimate the list price for each item. Then calculate the list price. (*Hint:* First use your calculator to find what 1% is worth.)

Sale Price	Percent of List Price	Estimated List Price	Calculated List Price
\$120.00	60%	\$180	\$200
\$100.00	50%		
\$8.00	32%		
\$255.00	85%		
\$77.00	55%		
\$80.00	40%		
\$9.00	60%		
\$112.50	75%		
\$450.00	90%		

3. Alan is walking to a friend's house. He covered $\frac{6}{10}$ of the distance in 48 minutes. If he continues at the same speed, about how long will the entire walk take?

Using a Unit Fraction or a Unit Percent to Find the Whole (cont.)

 4. 24 is $\frac{1}{2}$ of what number?
 5. $\frac{2}{5}$ is $\frac{1}{2}$ of what number?

 6. 27 is $\frac{3}{4}$ of what number?
 7. $\frac{3}{8}$ is $\frac{3}{4}$ of what number?

 8. 60 is 50% of what number?
 9. 16 is 25% of what number?

 10. 40 is 80% of what number?
 9. 16 is 25% of what number?

The problems below are from an arithmetic book published in 1906. Solve the problems.

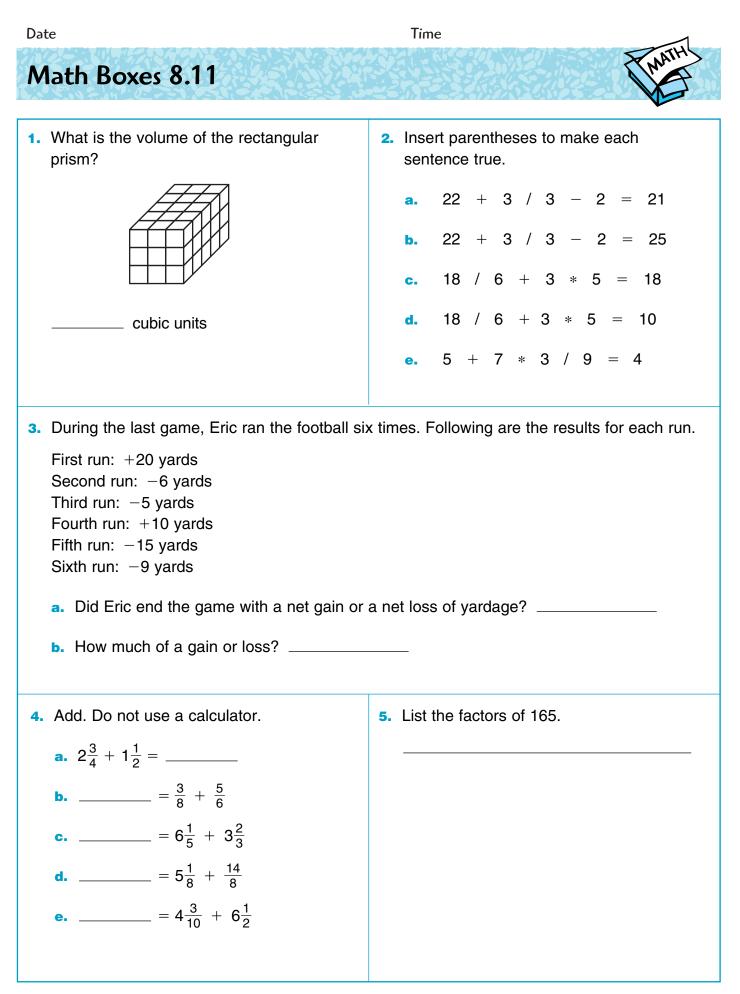
- 11. If the average coal miner works $\frac{2}{3}$ of a month of 30 days, how many days during the month does he work? _____ days
- **12.** A recipe for fudge calls for $\frac{1}{4}$ of a cake of chocolate. If a cake costs 20¢, find the cost of the chocolate called for by the recipe. _____ ¢
- **13.** In target practice the battleship *Indiana* shot at a target 24 times. If $\frac{3}{4}$ of the shots hit, how many successful shots were fired? _____ shots
- 14. A collection of mail that required 6 hours for a postman to make with a horse and wagon was made in an automobile in $\frac{5}{12}$ the time. How long did the automobile take? _____ hours
- **15.** How many corks per day does a machine in Spain make from the bark of the cork tree, if it makes $\frac{1}{3}$ of a sack of 15,000 corks in that time? _____ corks *Source: Milne's Progressive Arithmetic*

Challenge

16. It's easy to write 1 as a sum of unit fractions if the same unit fraction may be used more than once. For example: $\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} = 1$

Try to write 1 as a sum of unit fractions without repeating a fraction. Try to find more than one solution.

Date	Time
Math Boxes 8.10	(NATH)
11.1 Sto 2.11 (2.23) / 541 (2.23) (2.3)	
 Use your Geometry Template to draw a parallelogram. 	2. Write > or <. a. 50% $\frac{2}{3}$ b. 620 - 80 30 * 40
What are some other names for the figure you drew?	c. $\frac{7}{8}$ $\frac{1}{4} + \frac{2}{4}$ d. 20 * 19 20 ² e. 0.35 + 0.25 $\frac{1}{8} + \frac{1}{8}$
135 136	
3. Draw and label a 30° angle.	 Circle the numbers below that are evenly divisible by 6.
	148 293 762 1,050 984
SRB 190	SRB
5. Solve.	
a. 75 b. 425 <u>* 88</u> <u>* 68</u>	c. 759 <u>* 13</u> <u>* 185</u>
	SRB 19 20



Da	te			Time
С	lass Survey			
1.	How many people	live in your home?		
	0 1-2 people	0 3–5 people	0	6 or more people
2.	What language do	you speak at home?	?	
	0 English	() Spanish	0	Other:
3.	Are you right- or le	eft-handed?		
	0 right-handed	0 left-handed		
4.	C I	u lived at your curren	nt ao	ddress? (Round to the nearest year.)
5.	•	estions above. Tell w answer to the questio	-	someone you don't know might be ou picked.
6.	Fifteen percent of	the 20 students in M	s. S	wanson's class were left-handed.
	·	ts were left-handed?		
7.	Another 10% spea		spea	n Middle School speak English at home. ak other languages. About how many
	English:	_ students		
	Spanish:	_ students		
	Other:	students		
8.	The government re	eported that 5% of 90	0,00	0,000 workers do not have jobs.
	How many worker	s were jobless?		workers

Rural and Urban Populations

The U.S. Census Bureau classifies where people live according to the following rule: **Rural** areas are communities having fewer than 2,500 people each. **Urban** areas are communities having 2,500 or more people each.

1. According to the Census Bureau's definition, do you live in a rural or an urban area?

How did you decide? ______

the Census Bureau defines as urban. This was not always the case. When the United States was formed, it was a rural nation.

Work with your classmates and use the information in the *Student Reference Book,* pages 308, 309, and 334 to examine the transformation of the United States from a rural to an urban nation.

2. My group is to estimate the number of people living in	(rural or urban)	areas in
(1790, 1850, 1900, or 2000)		
3. The total U.S. population in was (1790, 1850, 1900, or 2000)		
4. Estimate: The number of people living in	areas in	
(1790, 1850, 1900, or 2000) Make sure your answer is rounded to the nearest 100,0	00.	
5. Our estimation strategy was		

Rural and Urban Populations (cont.)

6. Use the estimates from the groups in your class to complete the following table.

Estimated Rural and Urban Populations, 1790–2000						
Year	Estimated Rural Population	Estimated Urban Population				
1790						
1850						
1900						
2000						

7. Is it fair to say that for more than half our nation's history, the **majority** of the population lived in rural areas?

Vocabulary majority means more than one-half of a count

Explain your answer.

8. About how many times larger was the rural population in 2000 than in 1790?

9. About how many times larger was the urban population in 2000 than in 1790?

Challenge

10. In which decade do you think the urban population became larger than the rural population?

Division
Math Message
 How many 2-pound boxes of candy can be made from 10 pounds of candy? boxes
 How many ³/₄-pound boxes of candy can be made from 6 pounds of candy? boxes
3. Sam has 5 pounds of peanut brittle. He wants to pack it in $\frac{3}{4}$ -pound packages.
How many full packages can he make? full packages
Will any peanut brittle be left over? How much? pound
4. 0 1 2 3 4 5 6 INCHES
a. How many 2-inch segments are there in 6 inches? segments
b. How many $\frac{1}{2}$ -inch segments are there in 6 inches? segments
c. How many $\frac{1}{8}$ -inch segments are there in $\frac{3}{4}$ of an inch? segments
Common Denominator Division

One way to divide fractions uses common denominators:

Step 1 Rename the fractions using a common denominator.

Step 2 Divide the numerators.

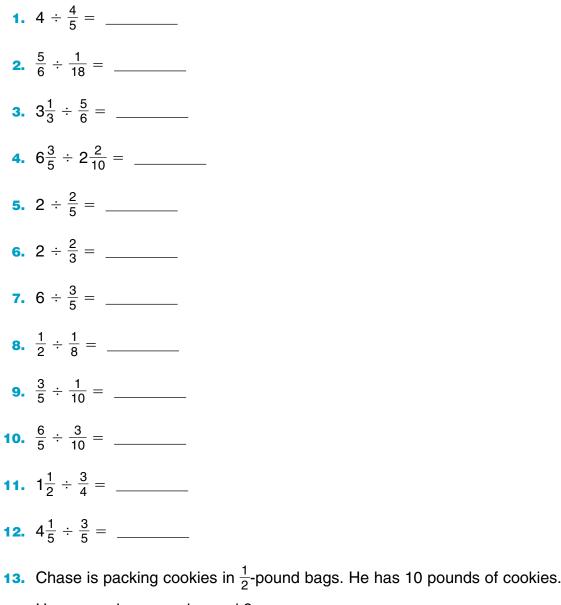
This method can also be used for whole or mixed numbers divided by fractions.

Examples

$$3 \div \frac{3}{4} = ?$$
 $\frac{1}{3} \div \frac{1}{6} = ?$ $3\frac{3}{5} \div \frac{3}{5} = \frac{18}{5} \div \frac{3}{5}$ $3 \div \frac{3}{4} = \frac{12}{4} \div \frac{3}{4}$ $\frac{1}{3} \div \frac{1}{6} = \frac{2}{6} \div \frac{1}{6}$ $= 18 \div 3 = 6$ $= 12 \div 3 = 4$ $= 2 \div 1 = 2$

Common Denominator Division (cont.)

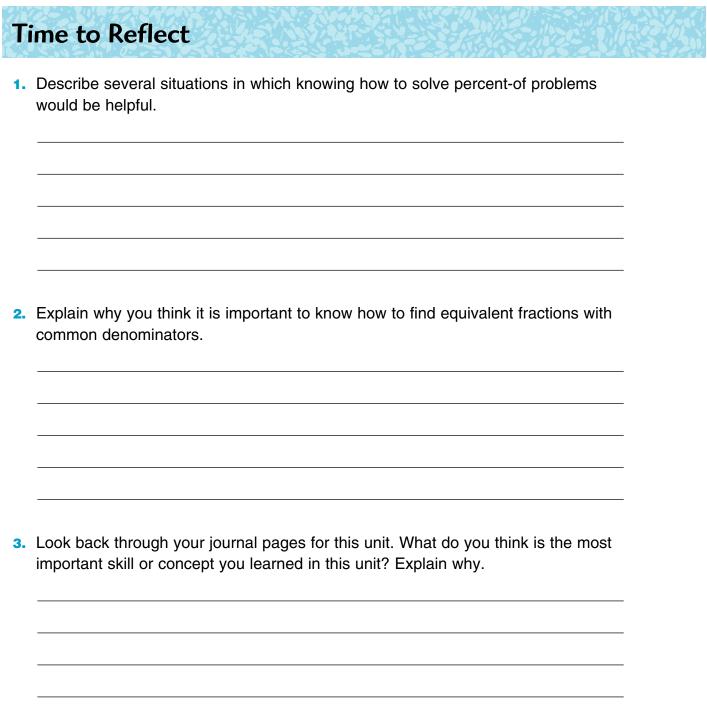
Solve.

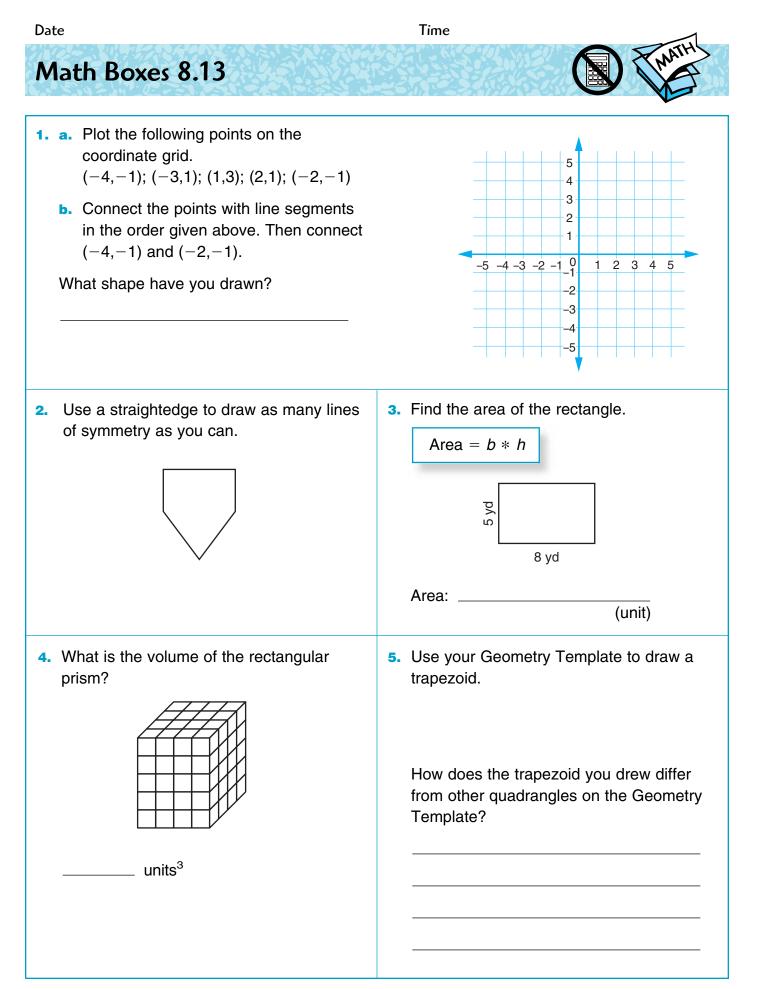


- How many bags can he pack? _____ bags
- 14. Regina is cutting lanyard to make bracelets. She has 15 feet of lanyard and needs $1\frac{1}{2}$ feet for each bracelet. How many bracelets can she make? _____ bracelets
- **15.** Eric is planning a pizza party. He has 3 large pizzas. He figures each person will eat $\frac{3}{8}$ of a pizza. How many people can attend the party, including himself?

_____ people

Date	Time
Math Boxes 8.12	I MATTA
 Use your Geometry Template to draw a triangle. What kind of triangle did you draw? 	2. Write > or <. a. $15 + 28 $ 10^2 b. $40 + 40 $ $3 * 30$ c. $\frac{1}{2} + \frac{1}{2} $ $\frac{3}{4}$ d. $\frac{19}{20}$ $0.6 + 0.3$ e. $55 \div 5$ $120 \div 12$
3. Draw and label a 170° angle.	 4. Circle the numbers below that are evenly divisible by 9. 3,735 2,043 192 769 594
5. Solve. a. 429 <u>* 15</u> <u>* 82</u>	c. 706 <u>* 189</u>





5

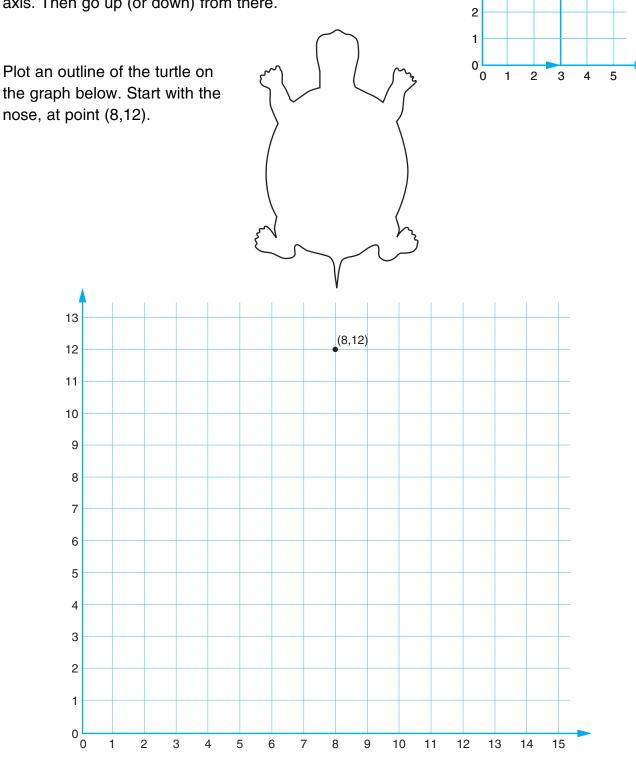
4

3

(3,4)

Plotting a Turtle

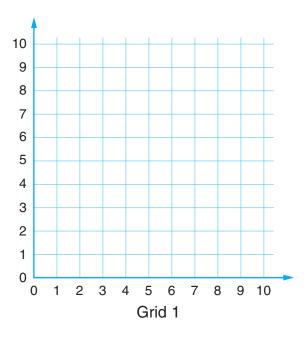
Points on a coordinate grid are named by ordered number pairs. The first number in an ordered number pair locates the point along the horizontal axis. The second number locates the point along the vertical axis. To mark a point on a coordinate grid, first go right (or left) on the horizontal axis. Then go up (or down) from there.



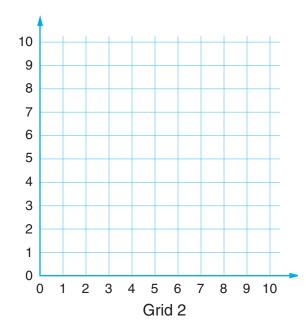
Hidden Treasure Gameboards

Each player uses Grids 1 and 2.

Grid 1: Hide your point here.

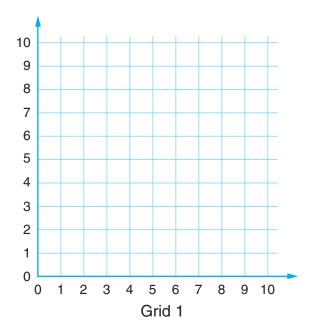


Grid 2: Guess other player's point here.

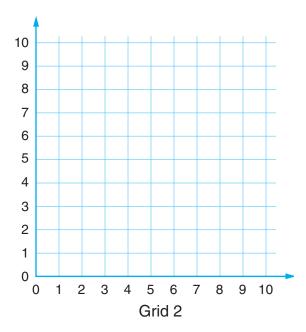


Use Grids 1 and 2 to play another game.

Grid 1: Hide your point here.



Grid 2: Guess other player's point here.



Date

Savings = 350 + (25 * number of weeks)

Savings = 350 - (25 * number of weeks)

Savings = \$350 * number of weeks

b. Meredith received \$350 for her birthday. She deposited the entire amount in the

c. Julian started a new savings account with \$50. Every week after that he deposited \$75.

2. Explain how you decided which graph matches each number story.

3. Circle the rule below that best fits the number story in Problem 1a above.

bank. Every week she withdrew \$50.

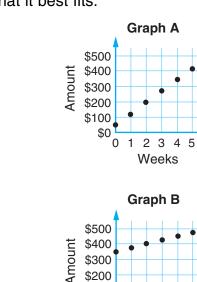
a. Juanita started with \$350. She saved

another \$25 every week.

1. Draw a line matching each graph below to the number story that it best fits.

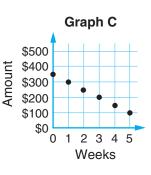
Time





\$100 \$0

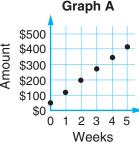
0 1

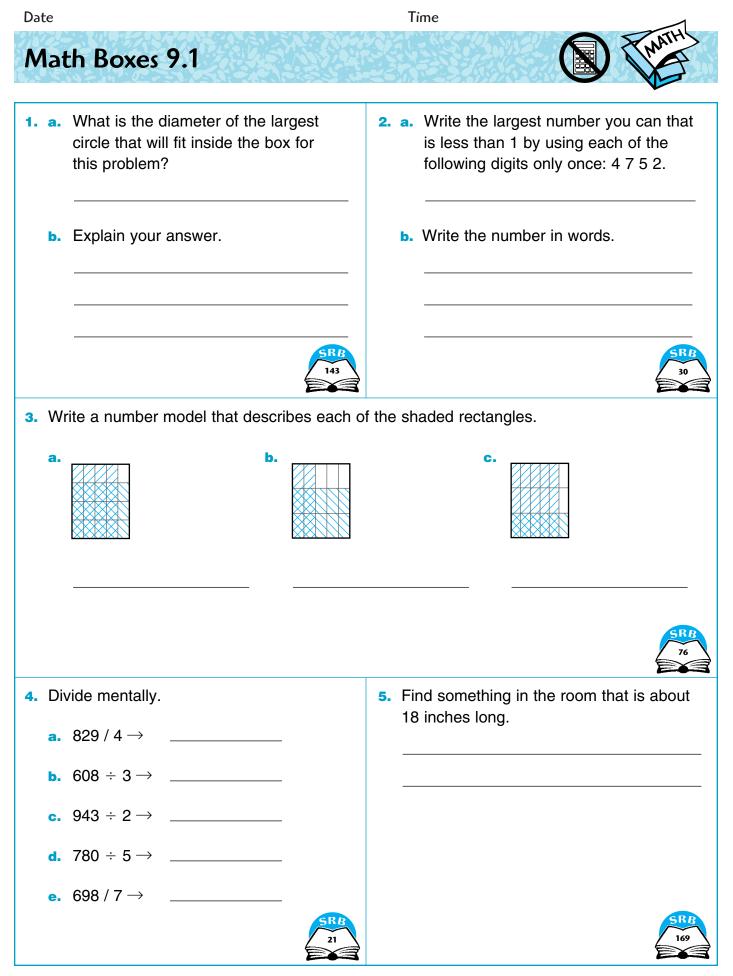


234

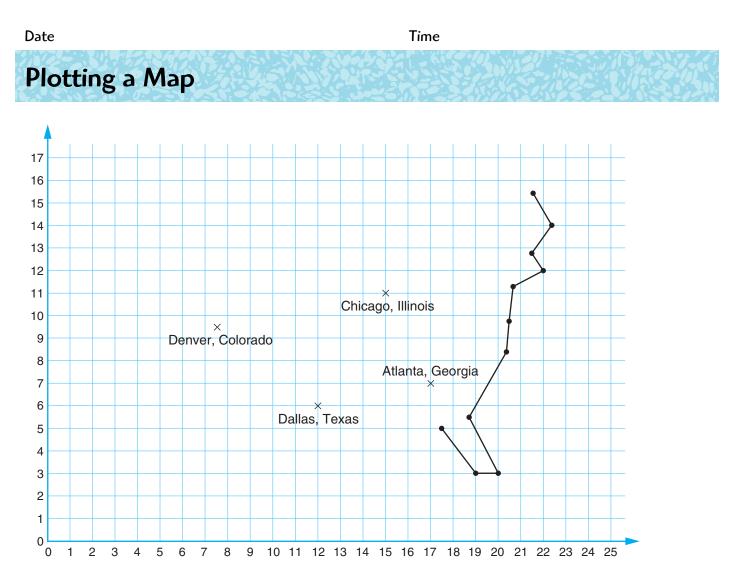
Weeks

5





Use with Lesson 9.1.



1. a. Plot the following ordered number pairs on the grid.

(21,14), (17,11), (17,13), (15,14), (2,16), (1,11), (2,8), (3,6), (7.5,5.5), (11,2.5), and (12.5,4)

- b. Connect all the points in the same order in which they were plotted. Also connect (12.5,4) to (17.5,5) and (21.5,15.5) to (21,14). When you have finished, you should see an outline map of the continental United States.
- 2. Write an ordered number pair to locate each city.
 - a. Chicago, Illinois (______, ____) b. Atlanta, Georgia (______, ____)
 - c. Dallas, Texas (______, ____) d. Denver, Colorado (______, ____)
- 3. Plot each city on the grid and write in the city name.
 - a. Billings, Montana (7.5,13) b. Salt Lake City, Utah (5.5,10.5)
- **4.** The U.S.–Mexican border is shown by line segments from (3,6) to (7.5,5.5) and from (7.5,5.5) to (11,2.5). Write the border name on the grid.

Use with Lesson 9.2.

Sailboat Graphs

- a. Using the ordered number pairs listed in the column titled Original Sailboat in the table below, plot the ordered number pairs on the grid titled Original Sailboat on the next page.
 - **b.** Connect the points in the same order that you plot them. You should see the outline of a sailboat.
- 2. Fill in the missing ordered number pairs in the last three columns of the table. Use the rule given in each column to calculate the ordered number pairs.

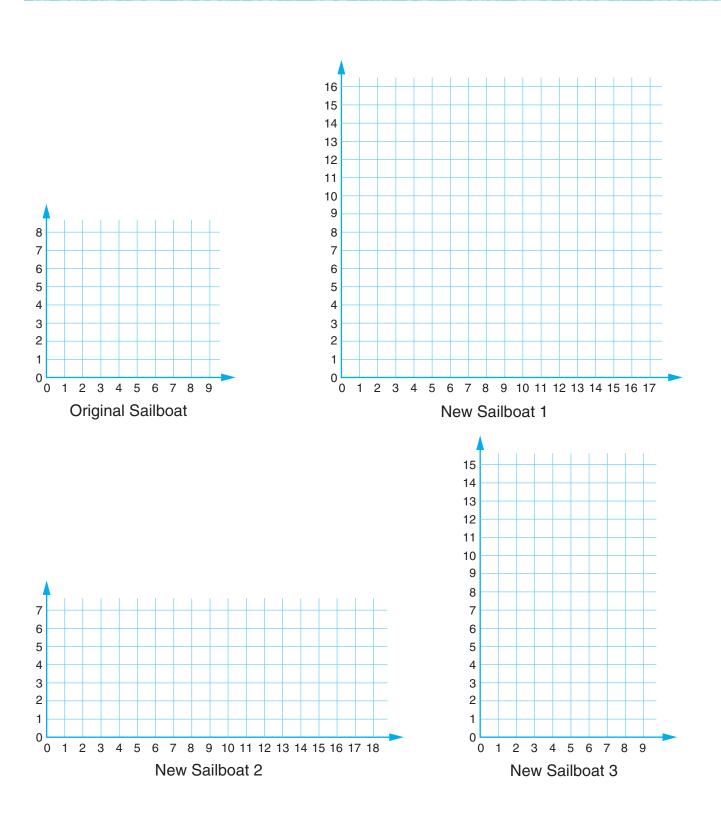
Original Sailboat	New Sailboat 1 Rule: Double each	New Sailboat 2 Rule: Double the first	New Sailboat 3 Rule: Double the	
	number of the original pair.	number of the original pair.	second number of the original pair.	
(8,1)	(16,2)	(16,1)	(8,2)	
(5,1)	(10,2)	(10,1)	(5,2)	
(5,7)	(10,14)	(10,7)	(5, 14)	
(1,2)	(,)	(,)	(,)	
(5,1)	(,)	(,)	(,)	
(0,1)	(,)	(,)	(,)	
(2,0)	(,)	(,)	(,)	
(7,0)	(,)	(,)	(,)	
(8,1)	(,)	(,)	(,)	

- **3. a.** Plot the ordered number pairs for New Sailboat 1 on the next page. Connect the points in the same order that you plot them.
 - **b.** Then plot the ordered number pairs for New Sailboat 2 and connect the points.
 - c. Finally, plot the ordered number pairs for New Sailboat 3 and connect the points.

Date

Time

Sailboat Graphs (cont.)



Place-Value Puzzles

1. The digit in the thousands place is 8.

The digit in the ones place is the sum of the digits in three centuries. (*Hint:* If there are ______ years in one century, then there are ______ years in three centuries.)

The digit in the millions place is $\frac{1}{10}$ of 40.

The digit in the hundred-thousands place is $\frac{1}{2}$ of the digit in the thousands place.

The digit in the hundreds place is the sum of the digit in the millions place and the digit in the ones place.

The rest of the digits are all 5s.

Write this number in words.	

2. The digit in the tenths place is 1.

The digit in the ones place is double the digit in the tenths place.

The digit in the thousandths place is $\frac{1}{3}$ of 21.

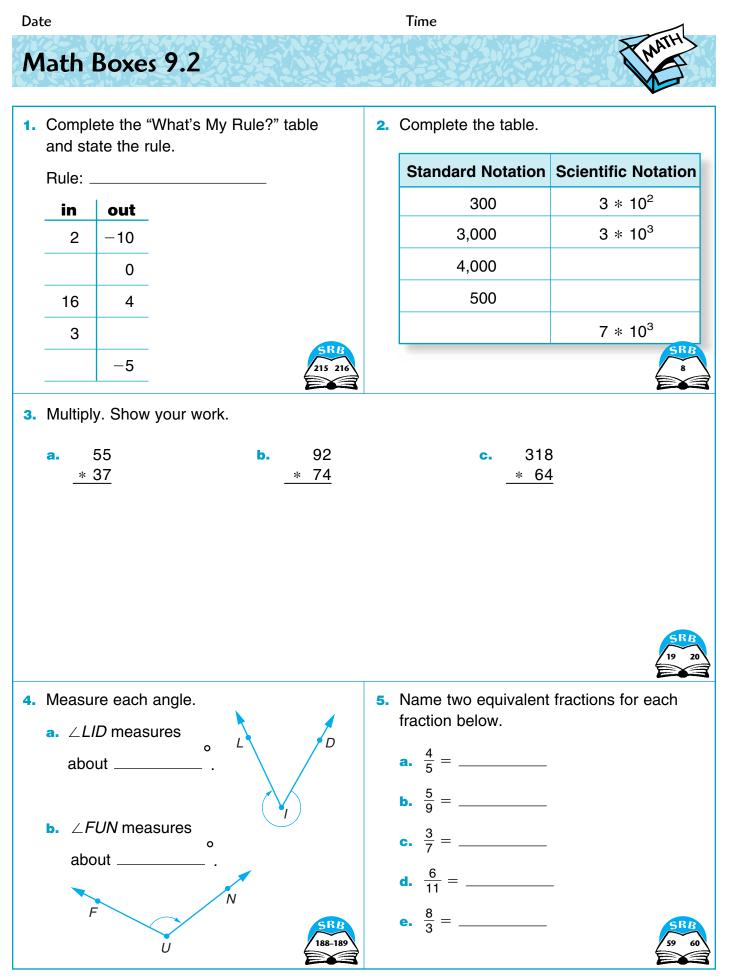
The digit in the hundreds place is three times the digit in the tenths place.

The digit in the ten-thousands place is an odd number less than 6 that you haven't used yet.

The rest of the digits are all 9s.

Write this number in words.

3. Make up a puzzle of your own.



Use with Lesson 9.2.

Date

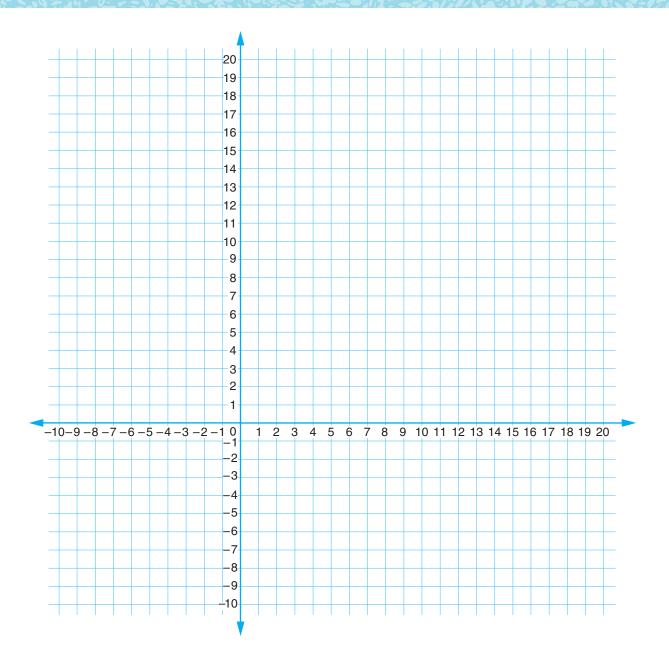
More Sailboat Graphs

- a. Using the ordered number pairs listed in the column titled Original Sailboat in the table below, plot the ordered number pairs on the grid on the next page.
 - **b.** Connect the points in the same order they were plotted. You should see the outline of a sailboat. Write "original" in the sail.
- 2. Fill in the missing ordered number pairs in the last three columns of the table. Use the rule given in each column to calculate the ordered number pairs.

Original Sailboat	New Sailboat 1	New Sailboat 2	New Sailboat 3	
	Rule: Add 10 to the first number of the original number pair.	Rule: Change the first number of the original pair to the opposite number.	Rule: Change the second number of the original pair to the opposite number.	
(9,3)	(19,3)	(-9,3)	(9, -3)	
(6,3)	(16, 3)	(-6,3)	(6, -3)	
(6,9)	(16,9)	(-6,9)	(6, -9)	
(2,4)	(,)	(,)	(,)	
(6,3)	(,)	(,)	(,)	
(1,3)	(,)	(,)	(,)	
(3,2)	(,)	(,)	(,)	
(8,2)	(,)	(,)	(,)	
(9,3)	(,)	(,)	(,)	

- a. Plot the ordered number pairs for New Sailboat 1 on the next page. Connect the points in the same order that you plot them. Write "1" in the sail.
 - Then plot the ordered number pairs for New Sailboat 2 and connect the points.
 Write "2" in the sail.
 - **c.** Finally, plot the ordered number pairs for New Sailboat 3 and connect the points. Write "3" in the sail.

More Sailboat Graphs (cont.)



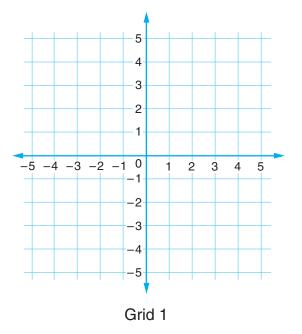
- **4.** Use the following rule to create a new sailboat figure on the grid above. Label it "4."
 - Rule: Add 10 to the second number of the original pair. Leave the first number unchanged.

Try to plot the new coordinates without listing them.

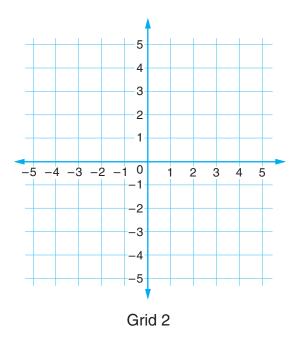
Advanced Hidden Treasure Gameboards

Each player uses Grids 1 and 2.

Grid 1: Hide your point here.

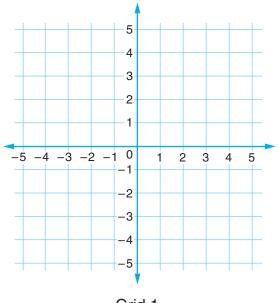


Grid 2: Guess other player's point here.



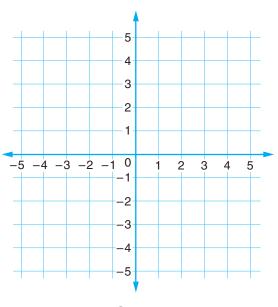
Use Grids 1 and 2 to play another game.

Grid 1: Hide your point here.



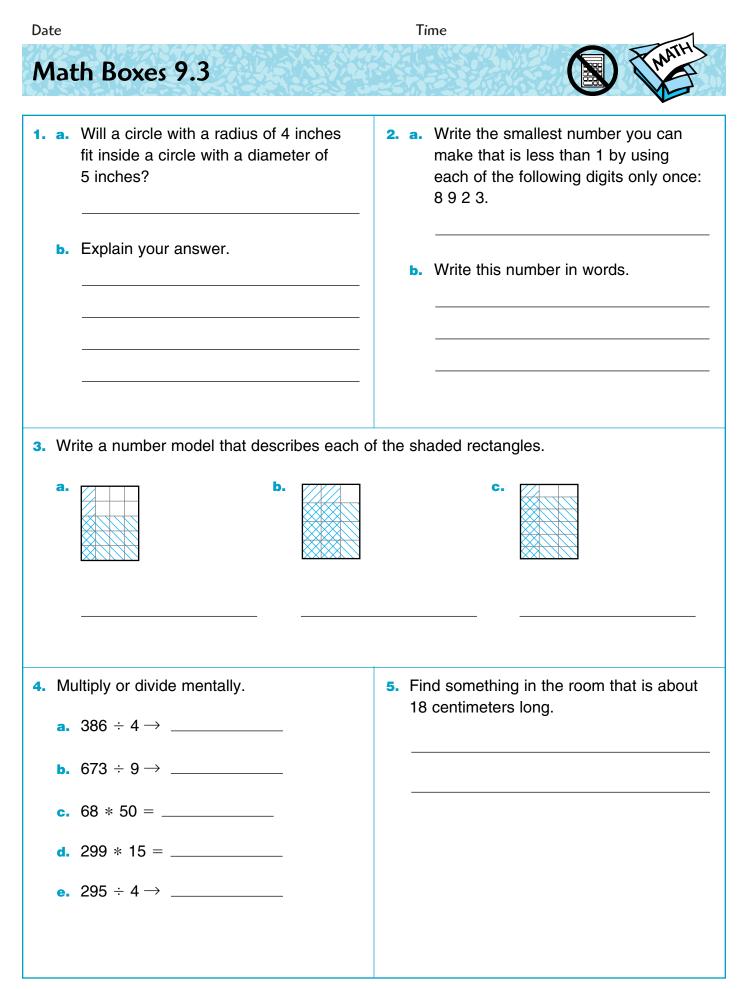
Grid 1

Grid 2: Guess other player's point here.



Grid 2

Date





$1 \text{ cm}^2 \qquad 1 \text{ cm}$ 1 cm		 height (or width) 		
	description → base → base → (or length)			
	B			
A				
	C			

1. Fill in the table. Draw rectangles D, E, and F on the grid.

Rectangle	Base (length)	Height (width)	Area
А	cm	cm	cm ²
В	cm	cm	cm ²
С	cm	cm	cm ²
D	6 cm	cm	12 cm ²
E	3.5 cm	cm	14 cm ²
F	3 cm	cm	10.5 cm ²

2. Write a formula for finding the area of a rectangle.

Area = _____

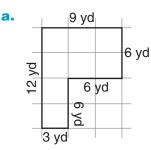
Area Problems

 A bedroom floor is 12 feet by 15 feet (4 yards by 5 yards).

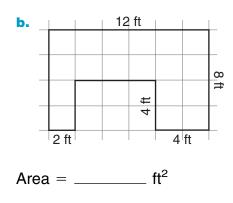
Floor area = _____ square feet

Floor area = _____ square yards

- 2. Imagine that you want to buy carpet for the bedroom in Problem 1. The carpet comes on a roll that is 6 feet (2 yards) wide. The carpet salesperson unrolls the carpet to the length you want and cuts off your piece. How long a piece will you need to cover the bedroom floor? ______
- Calculate the areas for the figures below.







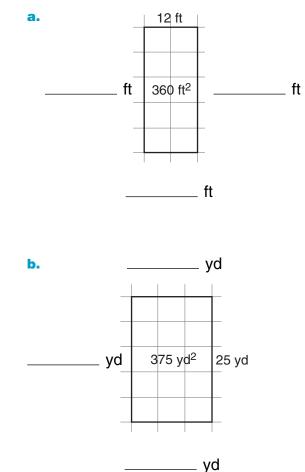
 Fill in the missing lengths for the figures below.

12 ft (4 yd)

6 ft

(2 yd)

15 ft (5 yd)

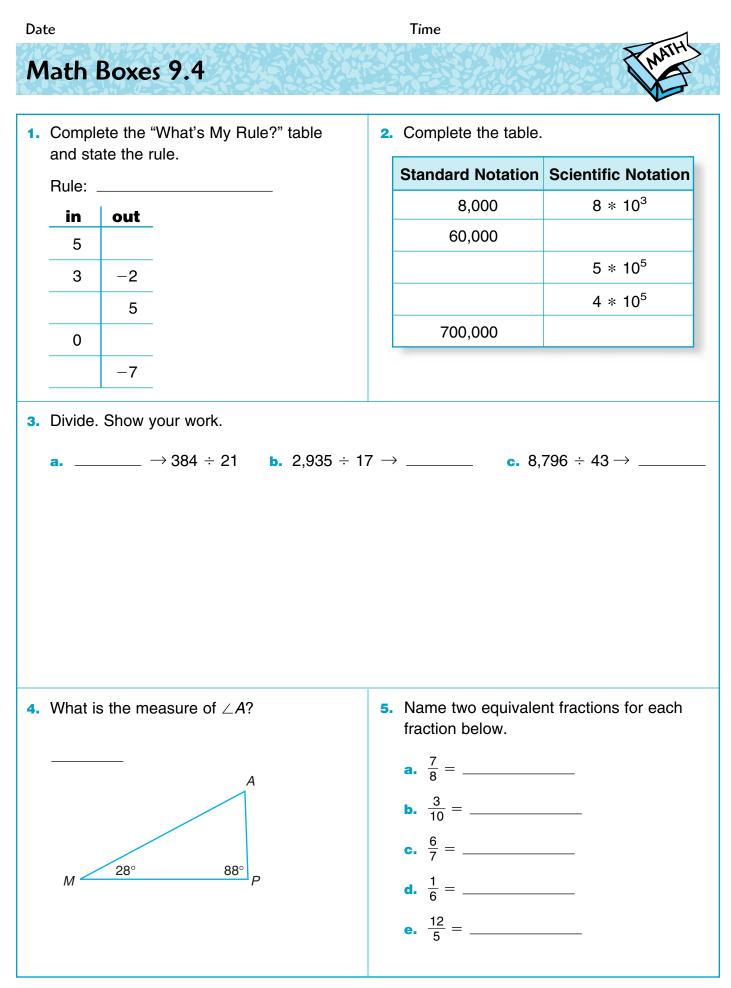


Review of 2-Dimensional Figures

Match each description of a geometric figure in Column A with its name in Column B. Not every name in Column B has a match.

Α В a. A polygon with 4 right angles octagon and 4 sides of the same length rhombus A polygon with 4 sides, no two of which need to be the same _____ right angle size _____ acute angle c. A quadrilateral with exactly one pair of opposite sides that are ____ trapezoid parallel _____ hexagon d. Lines in the same plane that never intersect _____ square e. A parallelogram (that is not a ____ equilateral triangle square) with all sides the same length perpendicular lines f. A polygon with 8 sides _ parallel lines g. Two intersecting lines that form a right angle pentagon h. A polygon with 5 sides _ isosceles triangle i. An angle that measures 90° ____ quadrilateral

j. A triangle with all sides the same length



Personal References

In *Fourth Grade Everyday Mathematics,* you found **personal references** for metric and U.S. customary units of length, weight, and capacity. These references are familiar objects whose sizes approximate standard measures. For example, for many people the distance across the tip of their smallest finger is about 1 centimeter.

Now you are working with **area**, so try to find personal references for area units.

Spend some time searching through your workspace or classroom to find common objects that have areas of 1 square inch, 1 square foot, 1 square yard, 1 square centimeter, and 1 square meter. The areas do not have to be exact, but they should be reasonable estimates. Ask a friend to look for references with you. Try to find more than one reference for each measure.

Unit	My Personal References
1 square inch (1 in. ²)	
1 square foot (1 ft ²)	
1 square yard (1 yd ²)	
1 square centimeter (1 cm ²)	
1 square meter (1 m ²)	

Personal References for Common Units of Area

Finding Areas of Nonrectangular Figures

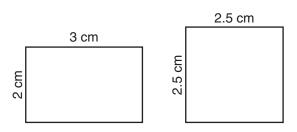
In the previous lesson, you calculated the areas of rectangular figures using two different methods.

• You counted the total number of unit squares and parts of unit squares that fit neatly inside the figure.

Date

			1 cm ²

 You used the formula A = b * h, where the letter A stands for area, the letter b for the length of the base, and the letter h for the height.

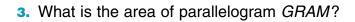


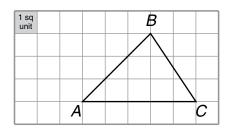
However, many times you will need to find the area of a figure that is not a rectangle. Unit squares will not fit neatly inside the figure, and you won't be able to use the formula for the area of a rectangle.

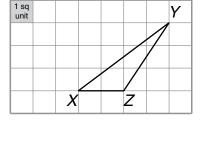
Working with a partner, think of a way to find the area of each of the figures below.

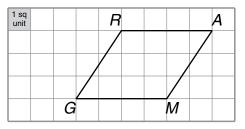
1. What is the area of triangle ABC?

2. What is the area of triangle *XYZ*?



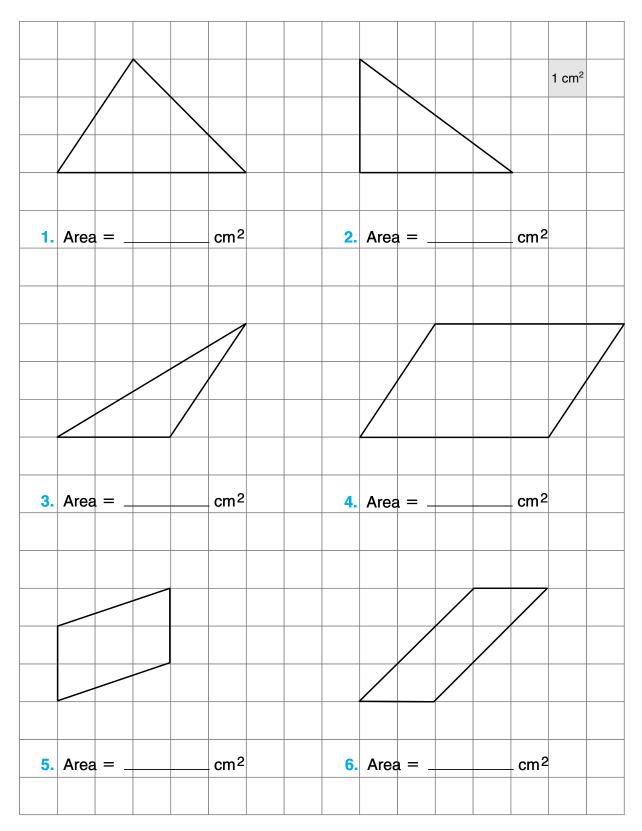


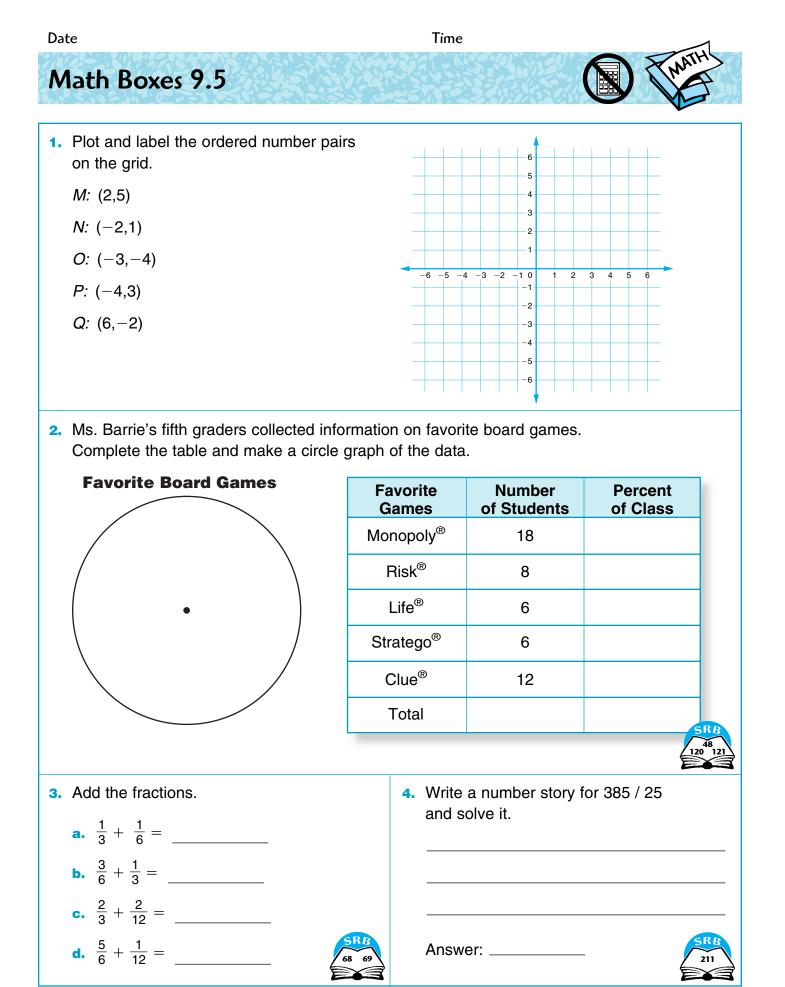




Areas of Triangles and Parallelograms

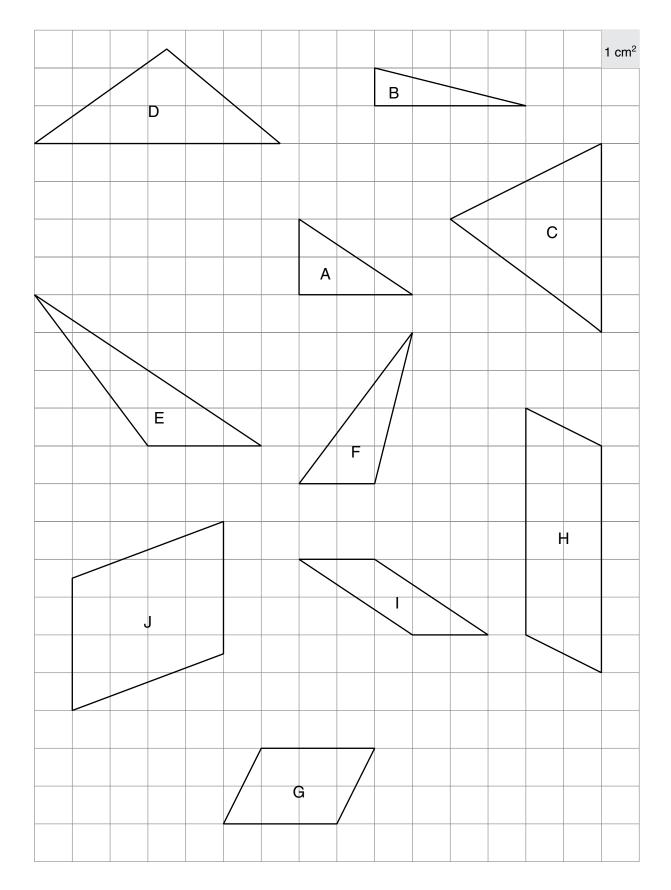
Use the rectangle method to find the area of each triangle and parallelogram below.





Use with Lesson 9.5.

The Rectangle Method



Finding Areas of Triangles and Parallelograms

1. Fill in the table. All figures are shown on journal page 318.

	Area	base	height	base * height		
Triangles						
A	3 cm ²	3 cm	2 cm	6 cm ²		
В	cm ²	cm	cm	cm ²		
С	cm ²	cm	cm	cm ²		
D	cm ²	cm	cm	cm ²		
Е	cm ²	3 cm	4 cm	cm ²		
F	cm ²	cm	cm	cm ²		
Parallelograms						
G	6 cm ²	3 cm	2 cm	6 cm ²		
Н	cm ²	cm	cm	cm ²		
I	cm ²	cm	2 cm	cm ²		
J	cm ²	cm	cm	cm ²		

2. Examine the results of Figures A–F. Propose a formula for the area of a triangle as an equation and as a word sentence. Discuss it with others.

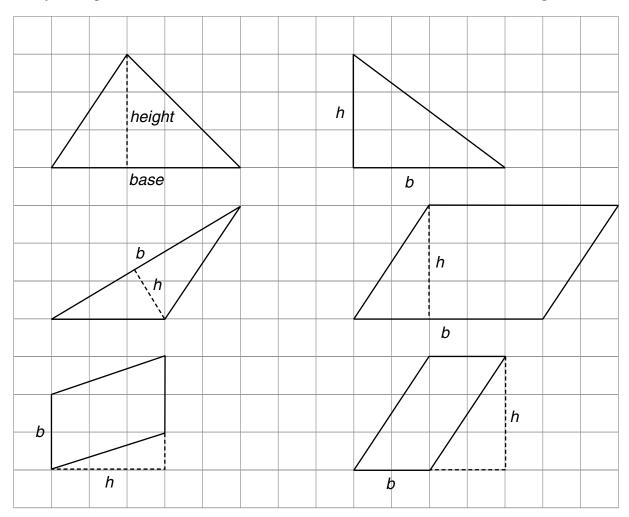
Area of a triangle = _____

3. Examine the results of Figures G–J. Propose a formula for the area of a parallelogram as an equation and as a word sentence. Discuss it with others.

Area of a parallelogram = _____

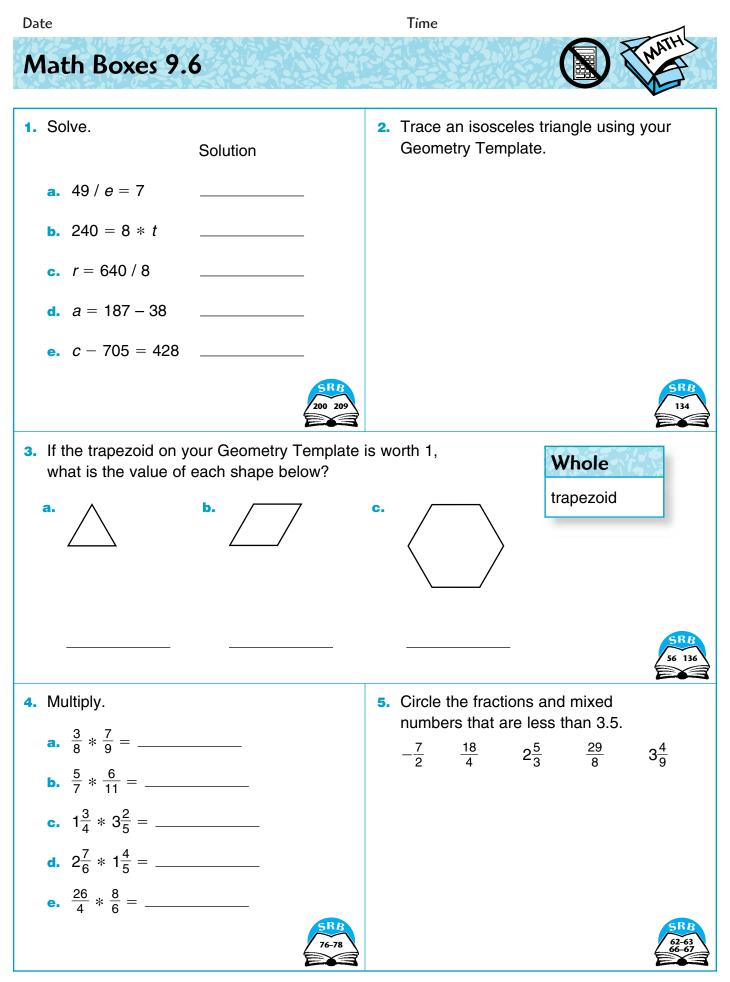
Defining Base and Height

Study the figures below. Then write definitions for the words base and height.



base:

height:



Earth's Water Surface

Math Message

Percent of Earth's surface that is covered by water:

My estimate:



A Sampling Experiment

My location is at latitude				and longitude	
My location is on	land	water.	(Circle one.)		

What fraction of the class has a water location?

Percent of Earth's surface that is covered by water:

My class's estimate: _____

Follow-Up

Your teacher can tell you the actual percent of Earth's surface that is covered by water, or you can look it up in a reference book.

Percent of Earth's surface that is covered by water:

Actual figure: _____

How does your class's estimate compare to the actual figure?

Note: This method of sampling usually gives results that are close to the actual value. However, it sometimes gives results that are very different.

Estimation Challenge: Area

What is the ground area of your school? In other words, what area of land is taken up by the ground floor?

Work alone or with a partner to come up with an estimation plan. How can you estimate the ground area of your school without measuring it with a tape measure? Discuss your ideas with your classmates.

My estimation plan:

My best estimate:

How accurate is your estimate? What range of areas might the actual area fall in?

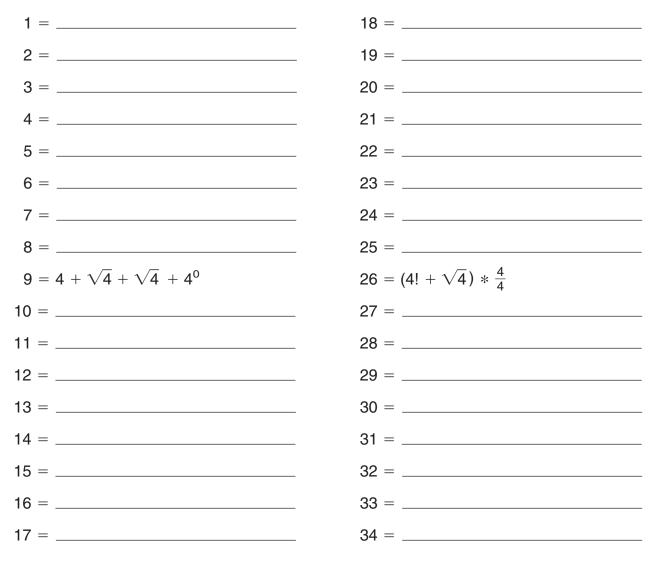
Date

The Four-4s Problem

Using only four 4s and any operation on your calculator, create expressions for values from 1 through 100. Do not use any other numbers except for the ones listed in the rules below. You do not need to find an expression for every number. Some are quite difficult. Try to find as many as you can today, but keep working when you have free time. The rules are listed below:

- You must use four 4s in every expression.
- You can use two 4s to create 44 or $\frac{4}{4}$. You can use three 4s to create 444.
- You may use 4^0 . $(4^0 = 1)$
- You may use $\sqrt{4}$. ($\sqrt{4} = 2$)
- You may use 4! (four factorial). (4! = 4 * 3 * 2 * 1 = 24)

Use parentheses as needed so that it is very clear what is to be done and in what order. Examples of expressions for some numbers are shown below.



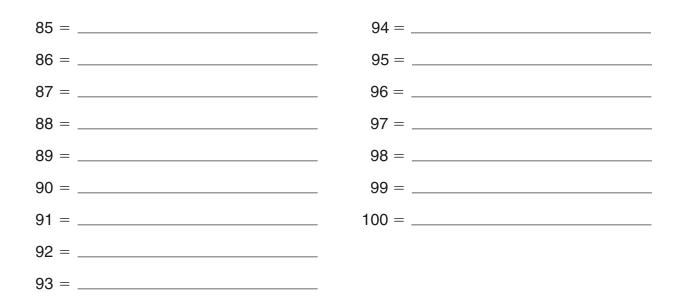
Use with Lesson 9.7.

The Four-4s Problem (cont.)

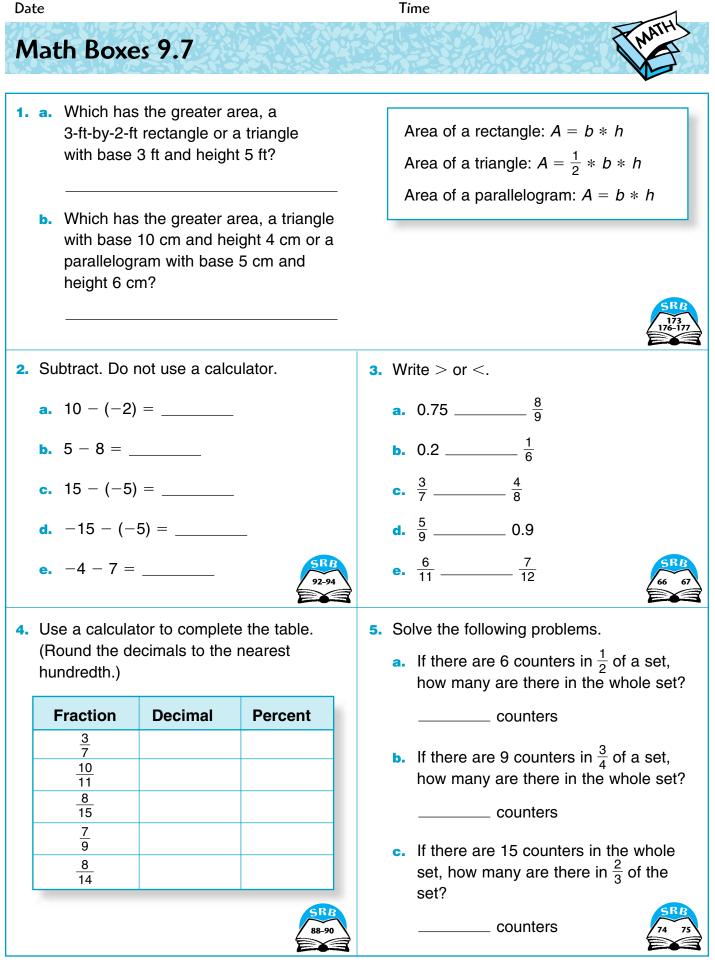
35 =
36 =
37 =
38 =
39 =
40 =
41 =
42 =
43 =
44 =
45 =
46 =
47 =
48 =
49 =
50 =
51 =
52 =
53 =
54 =
55 =
56 =
57 =
$58 = [(\sqrt{4} * (4! + 4)] + \sqrt{4}]$
59 =

60 =
61 =
62 =
63 =
64 =
65 =
66 =
67 =
68 =
69 =
70 =
71 =
72 =
73 =
74 =
75 =
76 =
77 =
78 =
79 =
80 =
81 =
82 =
83 =
84 =

The Four-4s Problem (cont.)



	Place-Value Magic If you follow the steps below, the results will always be 1,089.
DID	
Kou Or	Step 1 Pick any 3-digit number that has different digits in the ones place and the hundreds place.
	Step 2 Reverse the digits.
	Step 3 Find the difference between the number in Step 1 and the number in Step 2.
	Step 4 Reverse the digits of your difference in Step 3.
	Step 5 Find the sum of the numbers in Step 3 and Step 4.
	Your sum is 1,089.
	Example
	Step 1 427
	Step 2 724
	Step 3 724 - 427 = 297
	Step 4 792
	Step 5 792 + 297 = 1,089
	Can you figure out why this works?

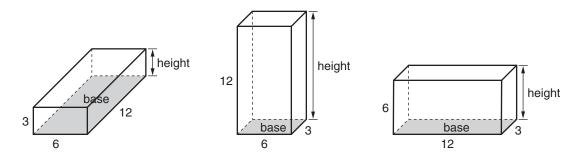




A **rectangular prism** is a geometric solid enclosed by six flat surfaces formed by rectangles. If each of the six rectangles is also a square, then the prism is a **cube**. The flat surfaces are called **faces** of the prism.

Bricks, paperback books, and most boxes are rectangular prisms. Dice and sugar cubes are examples of cubes.

Here are three different views of the same rectangular prism.



1. Study the figures above. Write your own definitions for base and height.

Base of a rectangular prism: ______

Examine the patterns on Activity Sheet 7. These patterns will be used to construct open boxes—boxes that have no tops. Try to figure out how many centimeter cubes are needed to fill each box to the top. Do not cut out the patterns yet.

- 2. I think that _____ centimeter cubes are needed to fill Box A to the top.
- 3. I think that ______ centimeter cubes are needed to fill Box B to the top.

Volumes of Rectangular Prisms

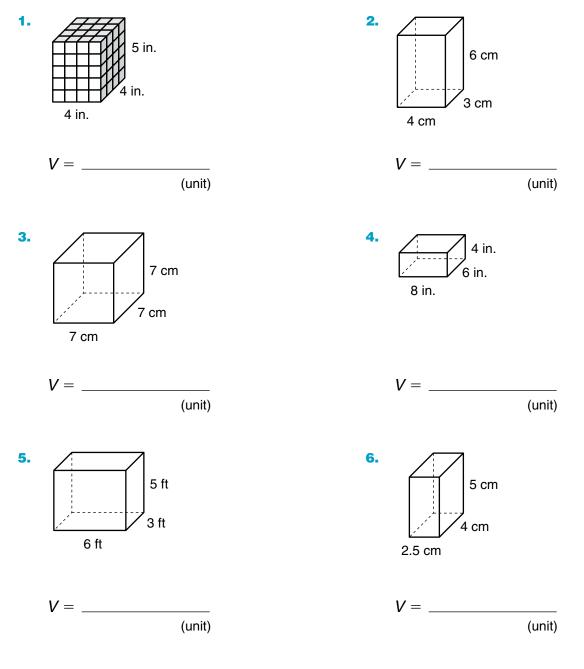
Write the formula for the volume of a rectangular prism.

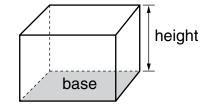
B is the area of the base.

h is the height from that base.

V is the volume of the prism.

Find the volume of each rectangular prism below.

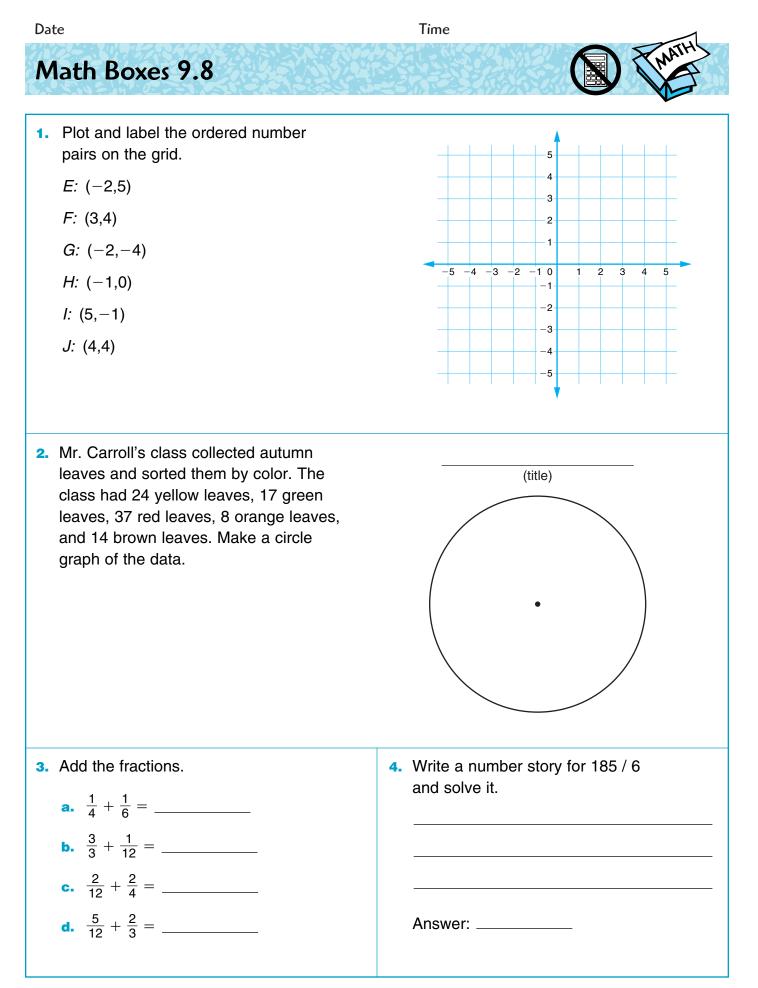




A Mental Calculation Strategy

When you are multiplying mentally, it is sometimes helpful to double one factor and halve the other factor.

Example 1 45 * 12 = ? **Step 1** Double 45 and halve 12: 45 * 12 = 90 * 6. **Step 2** Multiply 90 and 6: 90 * 6 = 540. *Example 2* 18 * 15 = ? **Step 1** Halve 18 and double 15: 18 * 15 = 9 * 30. **Step 2** Multiply 9 and 30: 9 * 30 = 270. *Example 3* 75 * 28 = ? Step 1 Double 75 to get 150 and halve 28 to get 14. Step 2 Double again to get 300 and halve again to get 7. **Step 3** 75 * 28 = 300 * 7 = 2,100. Use the doubling and halving strategy to calculate mentally. Solve the problems below. **1.** 35 * 14 = _____ 2. 16 * 25 = _____ New number sentence: New number sentence: **3.** 18 * 35 = _____ **4.** 15 * 44 = _____ New number sentence: New number sentence: **5.** 14 * 55 = _____ **6.** 75 * 24 = _____ New number sentence: New number sentence: New number sentence:

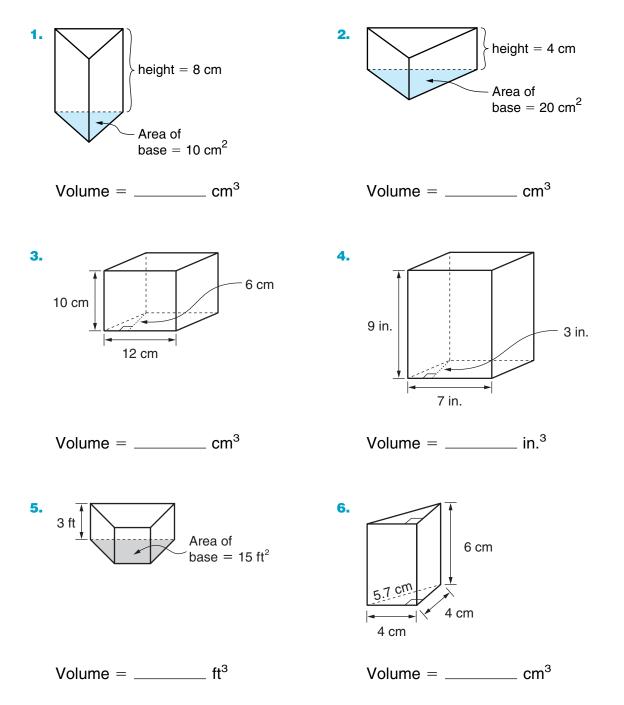


Time

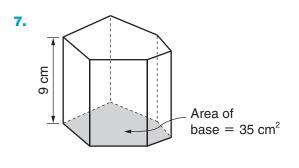
Volume of Prisms

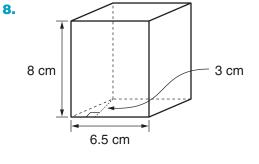
The volume *V* of any prism can be found with the formula V = B * h, where *B* is the area of the base of the prism, and *h* is the height of the prism for that base.

Find the volume of each prism.



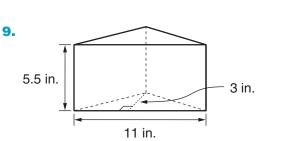
Volume of Prisms (cont.)

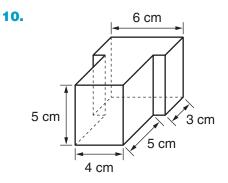


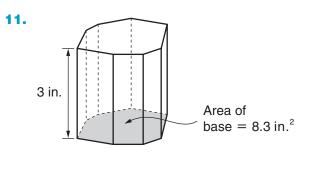


Volume = $_$ cm³



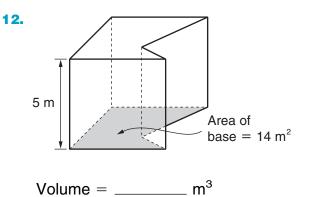


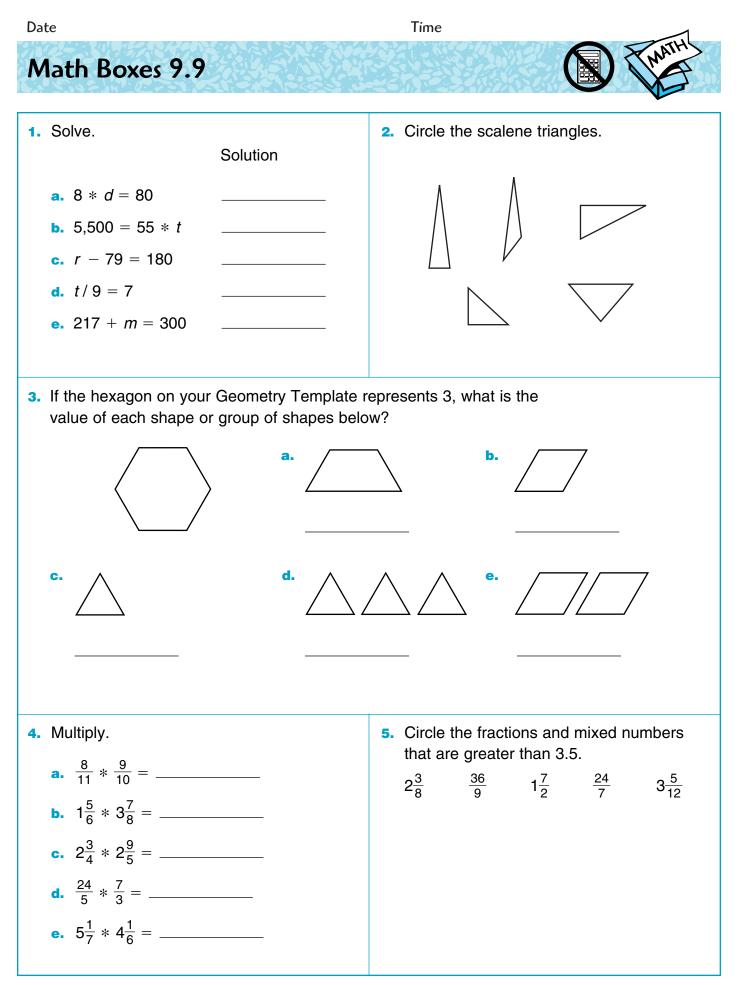




Volume = _____ in.³







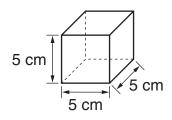
Units of Volume and Capacity

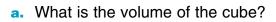
In the metric system, units of length, volume, capacity, and weight are related.

- The cubic centimeter (cm³) is a metric unit of volume.
- The liter (L) and milliliter (mL) are units of capacity.

1. Complete.

- a. 1 liter (L) = _____ milliliters (mL).
- **b.** There are _____ cubic centimeters (cm³) in 1 liter.
- **c.** So 1 cm³ = _____ mL.
- The cube in the diagram has sides 5 cm long.



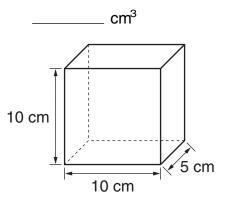


_____ cm³

b. If the cube were filled with water, how many milliliters would it hold?

_____ mL

a. What is the volume of the rectangular prism in the drawing?



b. If the prism were filled with water, how many milliliters would it hold?

_____ mL

____ L

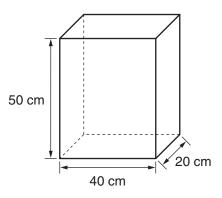
c. That is what fraction of a liter?

Complete.

- 4. 2 L = _____ mL
- **5.** $350 \text{ cm}^3 = ___ \text{mL}$
- 6. 1,500 mL = _____ L

Units of Volume and Capacity (cont.)

7. One liter of water weighs about 1 kilogram (kg).



If the tank in the diagram above is filled with water, about how much will the water

weigh? _____ kg

In the U.S. customary system, units of length and capacity are not closely related. Larger units of capacity are multiples of smaller units.

- 1 cup (c) = 8 fluid ounces (fl oz)
- 1 pint (pt) = 2 cups (c)
- 1 quart (qt) = 2 pints (pt)
- 1 gallon (gal) = 4 quarts (qt)
- **8. a.** 1 gallon = _____ quarts
 - **b.** 1 gallon = _____ pints
- **9. a.** 2 quarts = _____ pints
 - **b.** 2 quarts = _____ fluid ounces
- 10. Sometimes it is helpful to know that 1 liter is a little more than 1 quart. In the United States, gasoline is sold by the gallon. If you travel in Canada or Mexico, you will find that gasoline is sold by the liter. Is 1 gallon of gasoline more or less than 4 liters of gasoline?

Date



What are the dimensions of an open box—having the greatest possible volume—that can be made out of a single sheet of centimeter grid paper?

1. Use centimeter grid paper to experiment until you discover a pattern. Record your results in the table below.

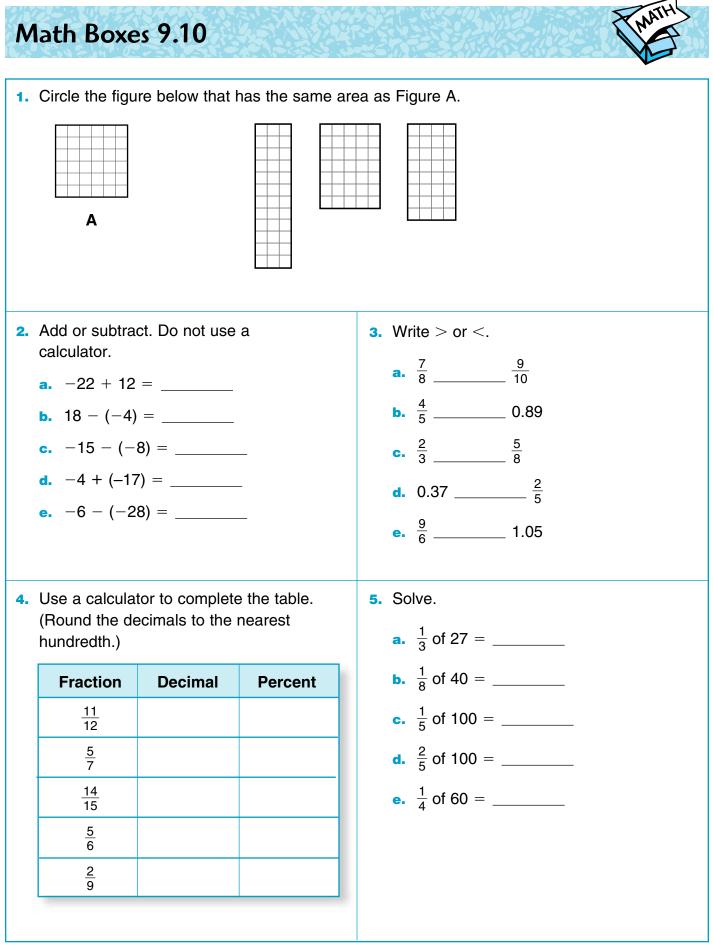
height of box	length of base	width of base	Volume of box
/ cm	20 cm	14 cm	
2 cm			
3 cm			

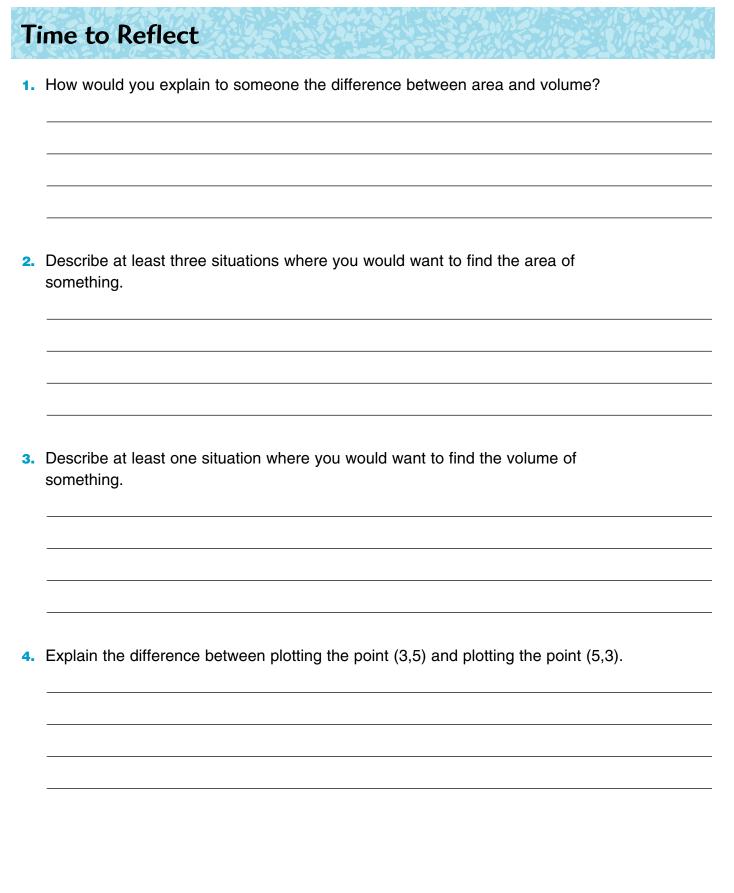
2. What are the dimensions of the box with the greatest volume?

height of box = _____ cm length of base = _____ cm

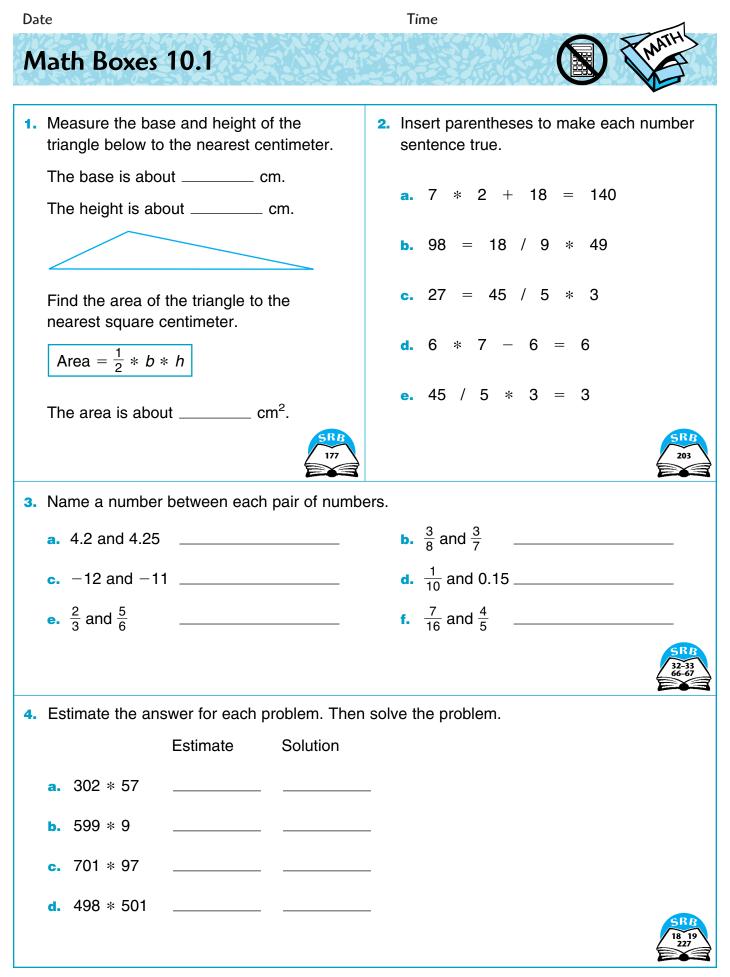
width of base = _____ cm Volume of box = _____ cm^3

Time





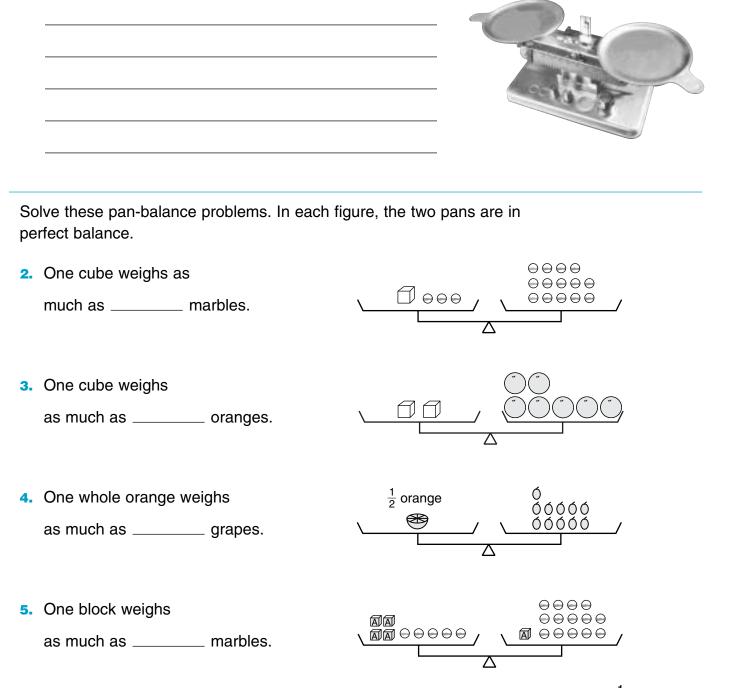
Date		Time
Math	Boxes 9.11	I MATHI
	-5) 4,-3)) -2)	
and sta	ete the "What's My Rule?" table te the rule. out 9 15 23 19	3. If the radius of a circle is 2.5 inches, what is its diameter? (unit) Explain.
4. Explain how you could find the area of the rectangle below.		5. Solve. Solution a. $\frac{3}{8} = \frac{a}{40}$ b. $-80 + c = 100$ c. $m * 25 = 400$ d. $s - 110 = -20$ e. $144 / z = 12$



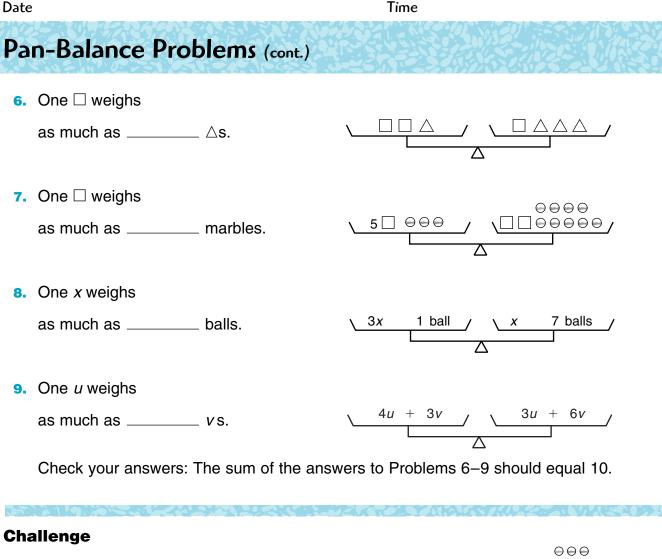
Pan-Balance Problems

Math Message

1. How would you use a pan balance to weigh an object?



Check your answers. The sum of the answers to Problems 2–5 should equal $39\frac{1}{2}$.



- **10.** An empty bottle weighs as much as 6 marbles.
 - The content of a full bottle weighs as much as _____ marbles.
- 00000 cmhalf both 00000 full / full 00000 Δ
- b. A full bottle weighs as much as _____ marbles.



Housing Boom

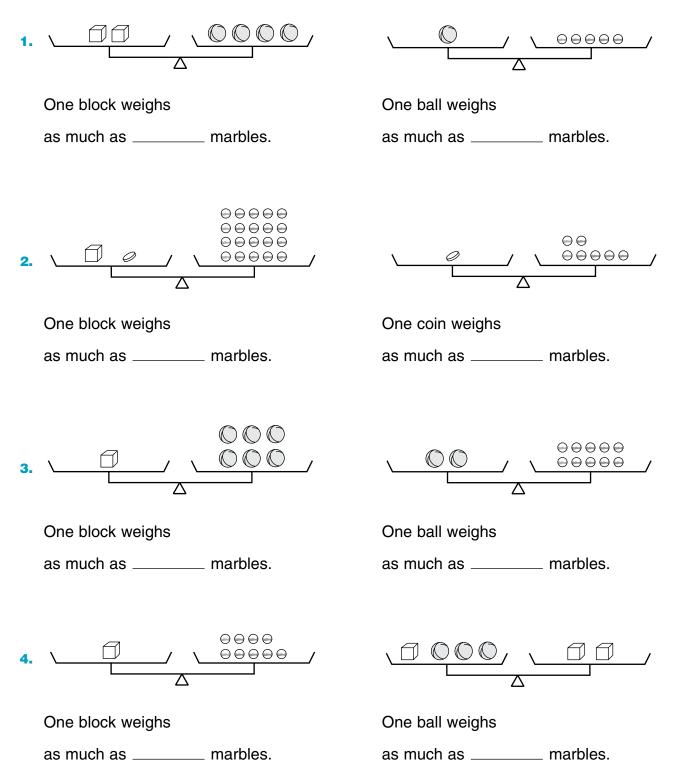
Some might say that Parker Brothers (the games company) has built more houses than any other developer in the world. Since 1935, the company has "built" more than 3 billion houses for its Monopoly[®] game.

Source: Games Magazine

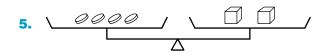
Time

More Pan-Balance Problems

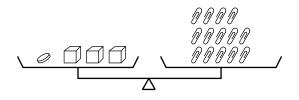
Solve these problems, using both pan balances in each problem. In each problem, the pans are in perfect balance. The weights of objects, such as blocks, balls, marbles, and coins, may be different from problem to problem, but are consistent within each problem.



More Pan-Balance Problems (cont.)

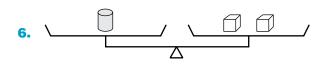


One coin weighs as much as _____ clips.



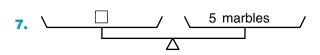


as much as _____ clips.



One can weighs

as much as _____ blocks.



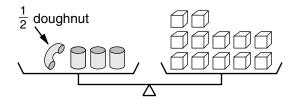
One 🗌 weighs

as much as _____ marbles.



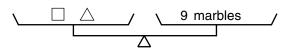
Each can weighs *B* ounces.

B = _____ ounces

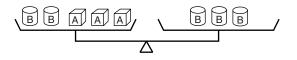


One doughnut weighs

as much as _____ blocks.

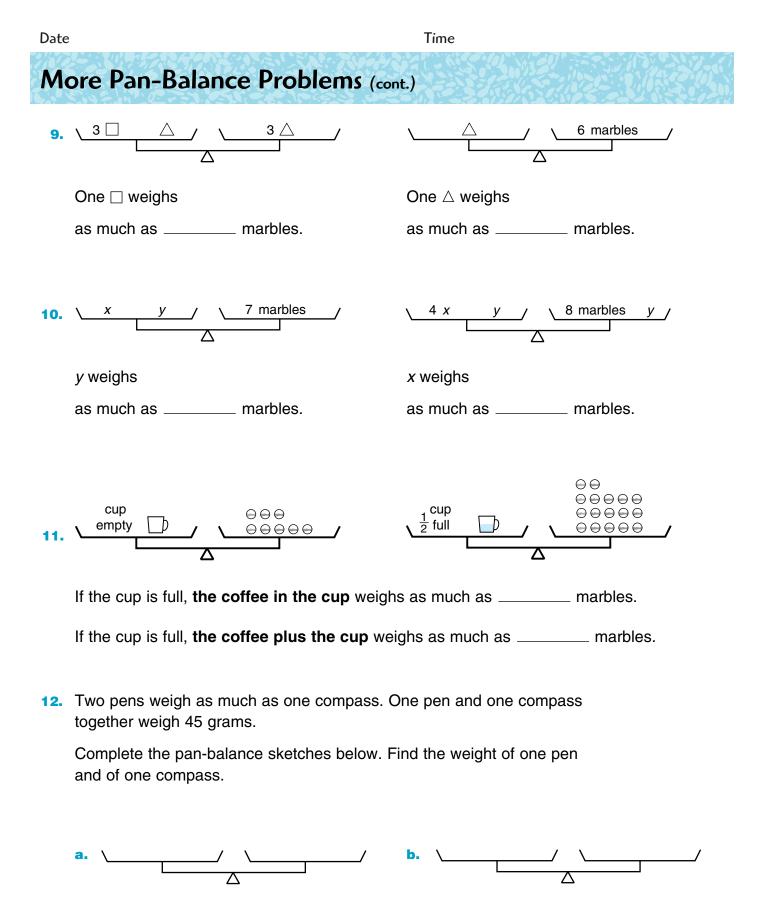


One \triangle weighs as much as _____ marbles.



Each cube weighs A ounces.

A = _____ ounces



One pen weighs _____ grams.

One compass weighs _____ grams.

Represent Number Stories

Circle each expression that correctly represents the information in the story. (There may be more than one answer.)

1. Melissa baked 5 trays with 12 cookies each. She sold 3 trays of 12 cookies.

Number of cookies sold:

(5 - 3) * 12 3 * 12 5 * 12

 Jonas was stocking up on soda for the family. He bought 8 six-packs of soda. His mom bought 3 more six-packs.

Total number of cans of soda:

(8 * 3) + 6 (8 * 3) + (8 * 6) 6 * (8 + 3)

3. Jenny bought 6 envelopes for 14 cents each and 6 stamps for 34 cents each.

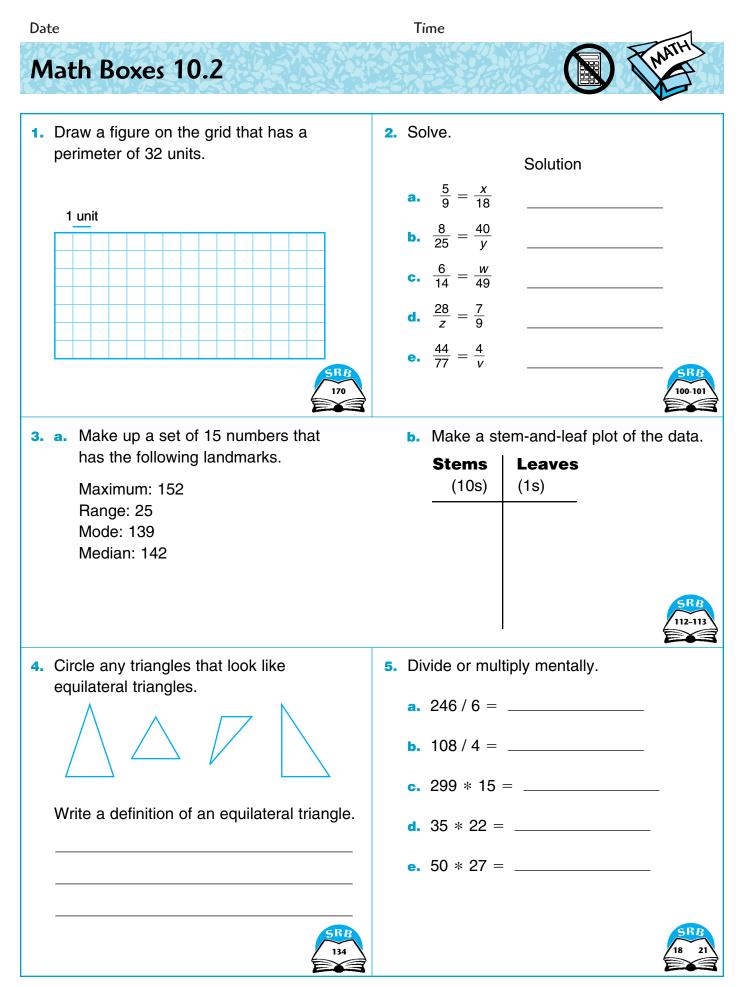
Amount of money spent:

- (6+6)*(14+34) (6*14)+(6*34) 6*(14+34)
- 4. Monty had 8 packages of 12 pencils when the year started. He used 3 packages of 12 during the school year.

Number of pencils left:

- (8-3) * 12 (8 * 12) (3 * 12) (8 + 3) * 12
- 5. Make up one of your own.

Your number expression: _____



Date	Time
Math Boxes 10.3	NATH!
1. Find the area and perimeter of the rectangle. 7 cm $3\frac{1}{2}$ cm	 2. Insert parentheses to make each number sentence true. a. 6 + 8 * 10 = 140 b. 21 = 42 / 6 - 4 c. 7 * 7 + 2 = 63
Area =(unit) Perimeter =(unit)	<pre>d. 3 * 15 - 3 = 36 e. 42 / 6 - 4 = 3</pre>
 3. Name a number between each pair of number a1.30 and -1.20	b. 8.05 and 8.10
4. Estimate the answer for each problem. Then Estimate Solution a. 60.3 * 71	- - -

Use with Lesson 10.3.

Algebraic Expressions

Date

Complete each statement below with an algebraic expression, using the suggested variable. The first problem has been done for you.

1. If Beth's allowance is \$2.50 more than Ann's, then Beth's allowance is

D + \$2.50

- 2. If John gets a raise of \$5 per week, then his salary is
 - \$_____.
- 3. If Ali's grandfather is 50 years older than Ali, then Ali is

_ years old.

4. Seven baskets of potatoes weigh

_ pounds.



John's salary is S dollars per week.



Ali's grandfather

is G years old.



A basket of potatoes weighs P pounds.







Ali





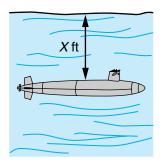
Ann's allowance is D dollars.



Algebraic Expressions (cont.)

5. If a submarine dives 150 feet, then it will be traveling at a depth

of ______ feet.



A submarine is traveling at a depth of *X* feet.

 The floor is divided up for gym classes into 5 equal-sized areas. Each class has a playing area of

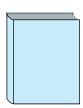
_____ ft².

 $\mathbb{D} \oplus \mathbb{C}$

The gym floor has an area of A square feet.

7. The charge for a book that is *D* days overdue is

_____ cents.



A library charges 10 cents for each overdue book. It adds an additional charge of 5 cents per day for each overdue book.

8. If Kevin spends $\frac{2}{3}$ of his allowance on a book, then he has

_____ dollars left.



Kevin's allowance is X dollars.

10.00		1911		
"W	'hat'	ς Μ	y Ru	e?"

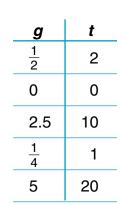
1. a. State in words the rule for the "What's My Rule?" table at the right.

Y
1
0
-5
-3
-2

- **b.** Circle the number sentence that describes the rule.
 - Y = X / 5 Y = X 4 Y = 4 X
- 2. a. State in words the rule for the "What's My Rule?" table at the right.

Q	Ζ
1	3
3	5
-4	-2
-3	-1
-2.5	-0.5

- **b.** Circle the number sentence that describes the rule.
 - Z = Q + 2 Z = 2 * Q $Z = \frac{1}{2}Q * 1$
- **3. a.** State in words the rule for the "What's My Rule?" table at the right.



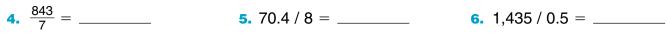
b. Circle the number sentence that describes the rule.

$$g = 2 * t$$
 $t = 2 * g$ $t = 4 * g$

Multiplication and Division Practice

Solve. Show your work in the space below the problems.

1. 384 * 1.5 = _____ **2.** 50.3 * 89 = ____ **3.** 824 * 75 = ____





C		
Speed a	and D	stance
Spece .		15

Math Message

1. A plane travels at a **speed** of 480 miles per hour. At that rate, how many miles will it travel in 1 minute? Write a number model to show what you did to solve the problem.

Number model:	Distance per minute:	_ miles

Rule for Distance Traveled

2. For an airplane flying at 8 miles per minute (480 mph), you can use the following rule to calculate the distance traveled for any number of minutes:

Distance traveled = 8 * number of minutes or d = 8 * t

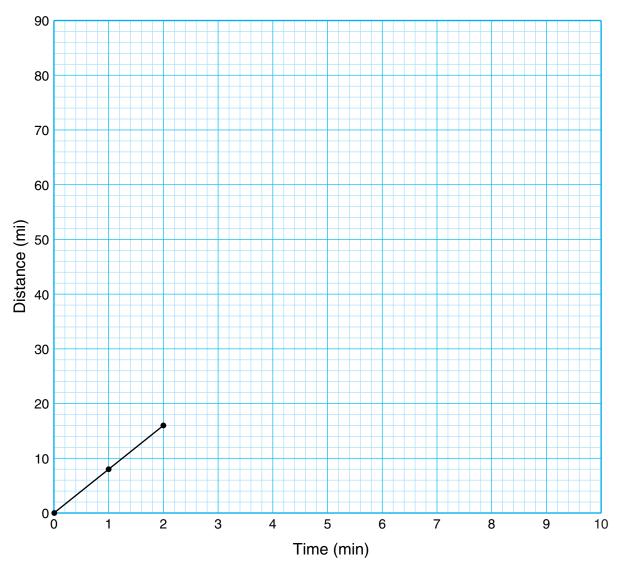
where *d* stands for the distance traveled and *t* for the time of travel, in minutes. For example, after 1 minute, the plane will have traveled 8 miles (8×1). After 2 minutes, it will have traveled 16 miles (8×2).

3. Use the rule d = 8 * tto complete the table at the right.

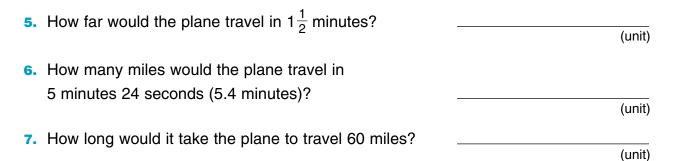
Time (min) (t)	Distance (mi) (8 * <i>t</i>)
1	8
2	16
3	
4	
5	
6	
7	
8	
9	
10	

Speed and Distance (cont.)

4. Complete the graph using the data in the table on page 354. Then connect the dots.



Use your graph to answer the following questions:



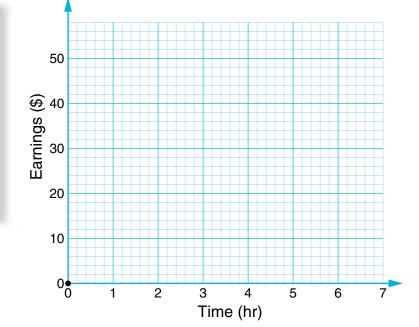
Representing Rates

Complete each table below. Then graph the data and connect the points.

1. a. Andy earns \$8 per hour. Rule: Earnings = \$8 * number of hours worked

Time (hr) (<i>h</i>)	Earnings (\$) (8 * <i>h</i>)
1	
2	
3	
	40
7	

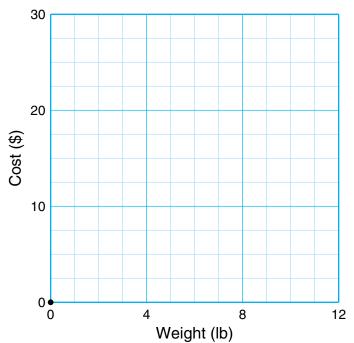
b. Plot a point to show Andy's earnings for $5\frac{1}{2}$ hours. How much would he earn?



2. a. Red peppers cost \$2.50 a pound. Rule: Cost = \$2.50 * number of pounds

Weight (lb) (<i>w</i>)	Cost (\$) (2.50 * <i>w</i>)
1	
2	
3	
	15.00
12	

b. Plot a point to show the cost of 8 pounds. How much would 8 pounds of red peppers cost?



Representing Rates (cont.)

3. a. Frank types an average of 45 words a minute.

Rule: Words typed = 45 * number of minutes

Time (min) (<i>t</i>)	Words (45 * <i>t</i>)	පු ³⁰⁰
1		§ 300 S 200
2		Je de la construction de
3		
	225	
6		Time (min)

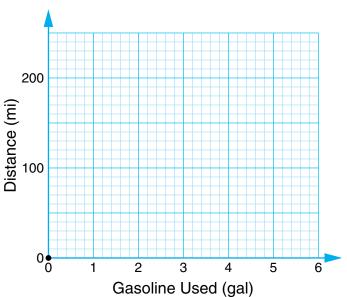
b. Plot a point to show the number of words Frank types in 4 minutes. How many words is that?

4. a. Joan's car uses 1 gallon of gasoline every 28 miles.

Rule: Distance = 28 * number of gallons

Gasoline (gal) (<i>g</i>)	Distance (mi) (28 * <i>g</i>)
1	
2	
3	
	140
$5\frac{1}{2}$	

 Plot a point to show how far the car would travel on 1.4 gallons of gasoline. How many miles would it go?



Area and Volume Review

Area and Volume Formulas

Area of rectangles and parallelograms

A = b * h, where A is the area, b is the length of the base, and h is the width or height

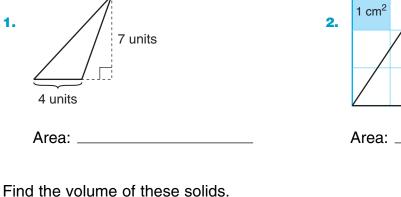
Area of triangles

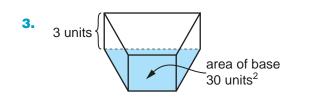
 $A = \frac{1}{2} * b * h$, where A is the area, b is the length of the base, and h is the height

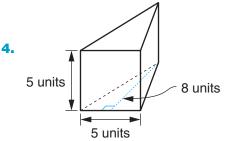
Volume of prisms

V = B * h, where B is the area of the base and h is the height

Find the area of these figures.

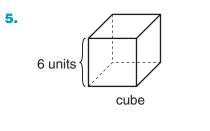






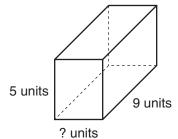
Volume:

6.

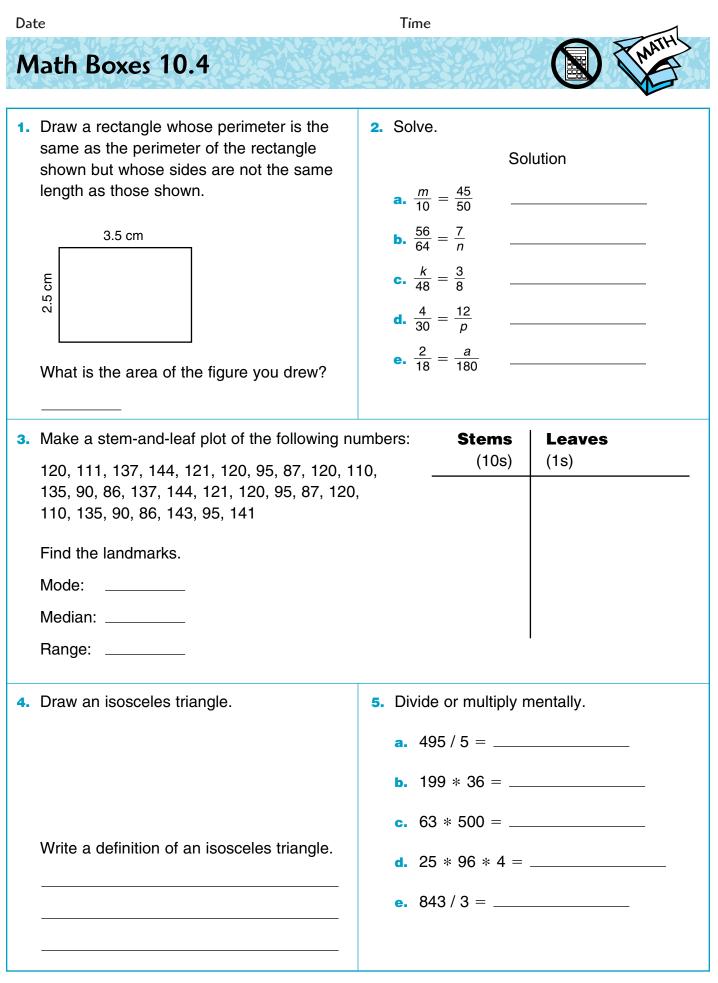


Volume: _





The volume of this prism is 180 cubic units. What is the width of its base?



Time

Predicting When Old Faithful Will Erupt Next

Old Faithful Geyser in Yellowstone National Park is one of nature's most impressive sights. Yellowstone has 200 geysers and thousands of hot springs, mud pots, steam vents, and other "hot spots"—more than any other place on Earth. Old Faithful is not the largest or tallest geyser in Yellowstone, but it is the most dependable geyser. Using the length of time of an eruption, park rangers can predict when the next eruption will begin.

Old Faithful erupts at regular intervals that are **predictable.** If you time the length of one eruption, you can **predict** about how long you must wait until the next eruption. Use this formula:

Waiting time = (10 * (length of eruption)) + 30 minutes W = (10 * E) + 30or W = 10E + 30

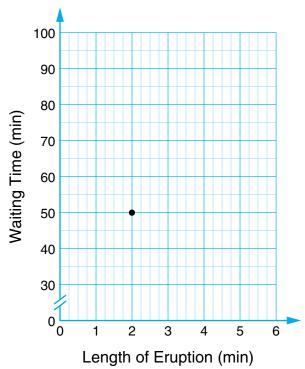
All times are in minutes.

1. Use the formula to complete the table below.

Length of Eruption (min) (<i>E</i>)	Waiting Time to Next Eruption (min) ((10 * <i>E</i>) + 30)
2 min	50 min
3 min	min
4 min	min
5 min	min
1 min	min
2 <u>1</u> min	min
3 min 15 sec	min
min	45 min

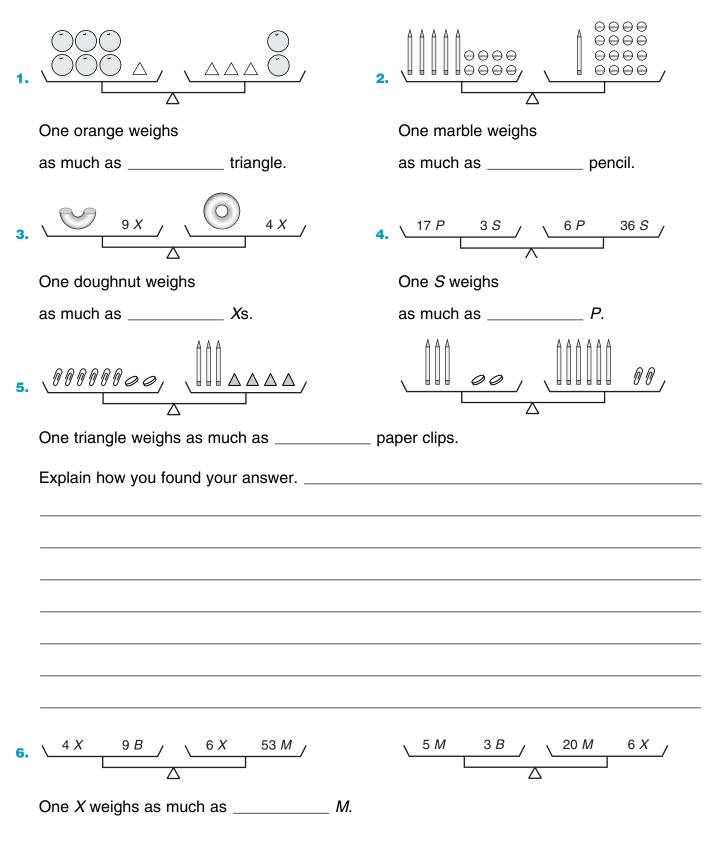
- It's 8:30 A.M., and Old Faithful has just finished a 4-minute eruption. About when will it erupt next?
- 4. The average time between eruptions of Old Faithful is about 75 minutes. So the average length of an eruption is about how many minutes?

 Graph the data from the table. One number pair has been plotted for you.



More Pan-Balance Practice

Solve these pan-balance problems. In each figure, the two pans are in perfect balance.



Math Boxes 10.5

- Use your Geometry Template to trace three kinds of triangles in the space below. Under each triangle, write what kind of triangle it is.
- 2. Add or subtract. a. -7 + (-3) =_____ b. 5 - (-8) =_____ c. -17 + 10 =_____ d. -15 - 15 =_____ e. 3 + (-20) =_____ 4. Add or subtract. a. $\frac{4}{5} + 1\frac{3}{8} =$ _____

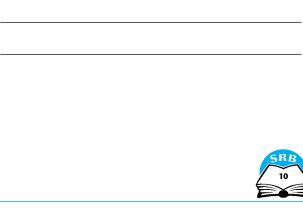
b. $1\frac{2}{4} - \frac{4}{5} =$ _____

c. $6\frac{3}{7} - 3\frac{1}{3} =$ _____

d. _____ = $4\frac{2}{9} + \frac{23}{6}$

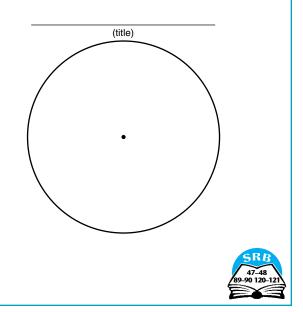


3. List all of the factors for 48.



 The table shows how Robert spent his allowance for the month of April. Complete the table and make a circle graph of the data.

Amount Spent	Percent of Allowance
\$2.50	
\$5.50	
\$0.50	
\$1.50	
	\$2.50 \$5.50 \$0.50



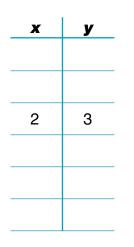
Use with Lesson 10.5.

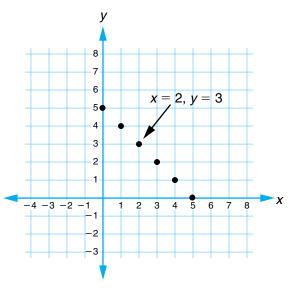
Time Date Math Boxes 10.6 2. Solve. **1.** Find the area of the parallelogram. Area of a parallelogram: A = b * h**a.** $\frac{1}{3}$ of 36 = _____ **b.** $\frac{2}{5}$ of 75 = _____ **c.** $\frac{3}{8}$ of 88 = _____ **d.** $\frac{5}{6}$ of 30 = _____ 1 unit 1 square unit e. $\frac{2}{7}$ of 28 = _____ Area: _____ square units 4. Complete the "What's My Rule?" table 3. Insert > or <. and state the rule. **a.** $\frac{9}{14}$ _____ $\frac{10}{3}$ Rule: _____ **b.** $\frac{6}{21}$ _____ $\frac{2}{6}$ in out **c.** $\frac{4}{11}$ _____ $\frac{7}{16}$ 8 -2 **d.** $\frac{8}{18}$ _____ $\frac{3}{7}$ 2 -6**e.** $\frac{5}{24}$ _____ $\frac{2}{10}$ 0 9 5. Solve. 215.29 128.07 b. 18.95 306.85 d. a. c. - 85.25 - 6.07 + 216.96+ 38.75

RI

Rules, Tables, and Graphs

1. Use the graph below. Find the *x*- and *y*-coordinates for each point shown. Then enter the *x* and *y* values in the table.





 Eli is 10 years old and can run an average of 5 yards per second. His sister Sara is 7 and can run an average of 4 yards per second.

Eli and Sara have a 60-yard race. Because Sara is younger, Eli gives her a 10-yard head start.

Complete the table showing the distances Eli and Sara are from the starting line after 1 second, 2 seconds, 3 seconds, and so on.

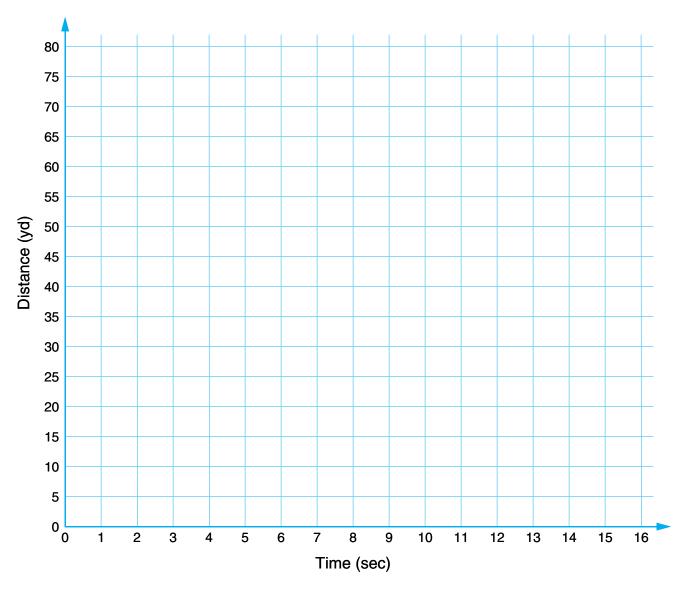
Use the table to answer the questions below.

- a. Who wins the race? ____
- **b.** What is the winning time?
- **c.** Who was in the lead during most of the race?

Time	Distance (yd)				
(sec)	Eli	Sara			
start	0	10			
1					
2		18			
3	15				
4					
5					
6					
7		38			
8					
9					
10					
11					
12					

Rules, Tables, and Graphs (cont.)

3. Use the grid below to graph the results of the race between Eli and Sara.



- 4. How many yards apart are Eli and Sara after 7 seconds?
- Suppose that Eli and Sara race for 75 yards instead of 60 yards.
 - a. Who would you expect to win?

 - c. How far ahead would the winner be at the finish line? ______ yards

Running and Walking Graph

Math Message

Rachel, William, and Tamara timed themselves traveling the same distance in different ways. Rachel ran, William walked, and Tamara walked toe-to-heel.

After they timed themselves, they drew a graph.

- Which line on the graph at the right is for Rachel? _______ A B C.
 Which line is for William? ______
 Which line is for Tamara? ______
- Sam came along later and was the slowest of all. He walked heel-to-toe backwards. Draw a line on the graph to show at what speed you think Sam walked.

Review: Algebraic Expressions

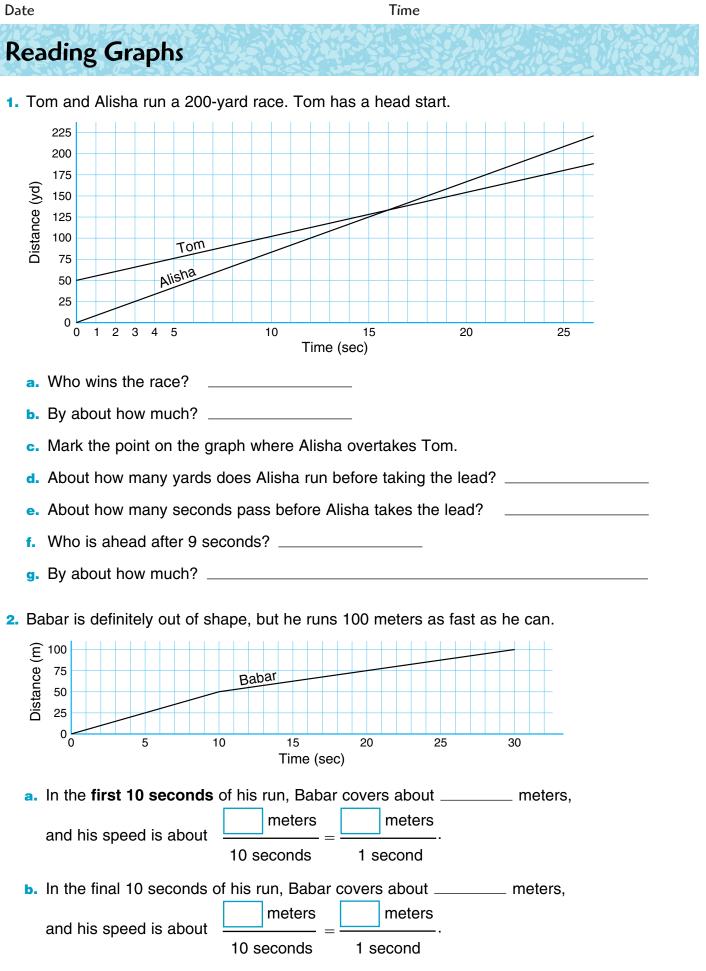
Complete each statement with an algebraic expression.

5. Bill is 5 years older than Rick. If Rick is *R* years old,

then Bill is _____ years old.

- Rebecca's piano lesson is one half as long as Lisa's. If Lisa's piano lesson is
 L minutes long, then Rebecca's is ______ minutes long.
- Jamie's dog weighs 3 pounds more than twice the weight of Eddy's dog.
 If Eddy's dog weighs *E* pounds, then Jamie's dog weighs ______ pounds.

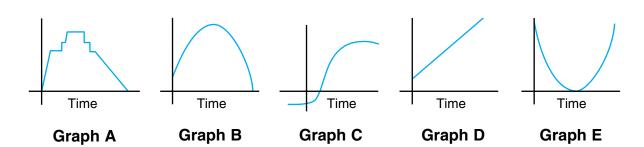
Time



Mystery Graphs

Date

Each of the events described below is represented by one of the following graphs:



Match each event with its graph.

 A frozen dinner is removed from the freezer. It is heated in a microwave oven. Then it is placed on the table.

Which graph shows the temperature of the dinner at different times?

 Satya runs water into his bathtub. He steps into the tub, sits down, and bathes. He gets out of the tub and drains the water.

Which graph shows the height of water in the tub at different times?

- 3. A baseball is thrown straight up into the air.
 - a. Which graph shows the height of the ball—from the time it is thrown until the time it hits the ground?
 - **b.** Which graph shows the speed of the ball at different times?

Graph _____

Graph _____

Graph _____

Graph _____

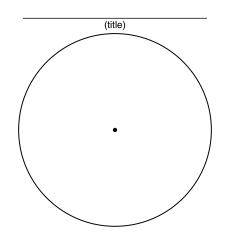
Math Boxes 10.7

1. a. I am a polygon with exactly 2. Add or subtract. 4 angles, each of a different size. **a.** 20 + (-10) = _____ What shape am I? **b.** -8 + (-17) = _____ b. Draw what I look like. **c.** -12 - (-12) =_____ **d.** -45 + 45 =e, -31 - 14 = _____ 4. Add or subtract. 3. List all of the factors for 144. **a.** $5\frac{4}{5} - 3\frac{7}{4} =$ _____ **b.** $3\frac{1}{8} + \frac{16}{6} =$ _____ **c.** _____ = $2\frac{4}{9} + 3\frac{7}{3}$ **d.** _____ = $1\frac{9}{10} - \frac{15}{8}$

Time

 Mr. Kim's art class asked 50 people each to name their favorite kind of movie. The results are shown in the table. Complete the table and then make a circle graph of the results.

Kind of Movie	Number of People	Percent of Total
Action	23	
Comedy	14	
Romance	2	
Thriller	7	
Mystery	4	
Total		



A Problem from the National Assessment

The following problem was in the mathematics section of a 1975 national standardized test.

Time

A square has a perimeter of 12 inches. What is the area of the square?

1. Your answer: _____ in.²

The table below gives the national results for this problem.

	13-Year-Olds	17-Year-Olds	Young Adults	
Correct answer	7%	28%	27%	
144 sq inches	12%	19%	25%	
48 sq inches	20%	10%	10%	
24 sq inches	6%	4%	2%	
12 sq inches	4%	3%	3%	
6 sq inches	4%	2%	1%	
3 sq inches	3%	2%	2%	
Other incorrect answers	16%	13%	10%	
No answer or "I don't know"	28%	19%	20%	

Explain why many students might have given the following answers:

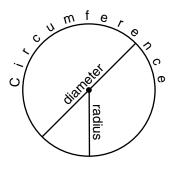
2. 144 square inches _____

3. 48 square inches _____

Ratio of Circumference to Diameter

You are going to explore the relationship between the circumference and the diameter of a circle.

 Using a metric tape measure, carefully measure the circumference and diameter of a variety of round objects. Measure to the nearest millimeter (one-tenth of a centimeter).



- 2. Record your data in the first three columns of the table below.
- **3.** In the fourth column, write the ratio of the circumference to the diameter as a fraction.
- **4.** In the fifth column, write the ratio as a decimal. Use your calculator to compute the decimal and round your answer to two decimal places.

			Ratio of Circumference to Diam			
Object	Circumference (C)	Diameter (<i>d</i>)	Ratio as a Fraction $(\frac{C}{d})$	Ratio as a Decimal (from calculator)		
Coffee cup	252 mm	80 mm	<u>252</u> 80	3./5		
	mm	mm				
	mm	mm				
	mm	mm				
	mm	mm				
	mm	mm				

- 5. What is the median of the circumference to diameter ratios in the last column?
- 6. The students in your class combined their results in a stem-and-leaf plot. Use that plot to find the class median value for the ratio $\frac{C}{d}$.

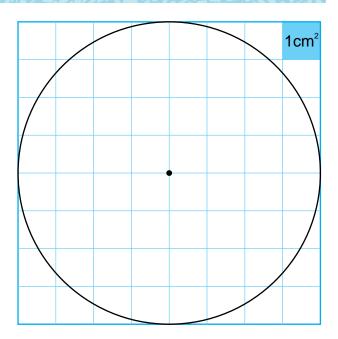
Date	Time
Math Boxes 10.8	C Thursday
1. Find the area of the triangle. Area of a Triangle: $A = \frac{1}{2} * b * h$ $Area of a Triangle: A = \frac{1}{2} * b * h$	2. If a set has 48 objects, how many objects are there in a. $\frac{3}{8}$ of the set? b. $\frac{8}{3}$ of the set? c. $\frac{5}{6}$ of the set? d. $\frac{7}{12}$ of the set? e. $\frac{17}{16}$ of the set?
Area: square units 3. Insert > or <. a. $\frac{8}{9}$ $\frac{8}{10}$ b. $\frac{3}{5}$ $\frac{3}{7}$ c. $\frac{6}{7}$ $\frac{5}{6}$ d. $\frac{7}{12}$ $\frac{7}{14}$ e. $\frac{9}{11}$ $\frac{14}{15}$	4. Complete the "What's My Rule?" table and state the rule. Rule:
 5. Solve. a. 40.017 b. 24.303 + 269.000 + 5.700 	c. 402.03 d. 590.32 <u>- 24.70</u> <u>- 465.75</u>

Measuring the Area of a Circle

Math Message

Use the circle at the right to solve Problems 1-4.

- The diameter of the circle is about _____ centimeters.
- The radius of the circle is about _____ centimeters.



- 3. The circumference of the circle is about _____ centimeters.
- 4. Find the area of this circle by counting squares. About _____ cm²

Follow-Up

5. What is the median of all the area measurements in your class? _____ cm²



More Pi, Anyone?

In 1999, Japanese computer scientists claimed a world record when they calculated pi to more than 206,158,430,000 digits on a computer at the University of Tokyo. The work took 13 hours to do and 46 hours to check. If a number with that many digits was printed on one line with 6 digits per centimeter, it would stretch more than 340,000 kilometers, or almost as far as the distance between Earth and the moon.

Source: University of Tokyo

Areas of Circles

Work with a partner. Use the same objects, but make separate measurements so that you can check each other's work.

- 1. Trace several round objects onto the grid on *Math Masters*, page 2.
- 2. Count square centimeters to find the area of each circle.
- **3.** Use a ruler to find the radius of each object. (*Remember:* The radius is half the diameter.) Record your data in the first three columns of the table below.

Object	Area	Radius	Ratio of Area to Radius Square		
	(sq cm)	(cm)	as a Fraction <i>A</i>	as a Decimal	
			$\frac{A}{r^2}$		

- 4. Find the ratio of the area to the square of the radius for each circle. Write the ratio as a fraction in the fourth column of the table. Then use a calculator to calculate the ratio as a decimal. Round the decimal to two decimal places and write it in the last column.
- 5. Find the median of the ratios in the last column.

Time

A Formula for the Area of a Circle

Your class just measured the area and radius of many circles and found that the ratio of the area to the square of the radius is about 3.

This was no coincidence: Mathematicians proved long ago that the ratio of the area of a circle to the square of its radius is always equal to π . This can be written as:

 $\frac{A}{r^2} = \pi$

Usually this fact is written in a slightly different form, as a formula for the area of a circle.

The formula for the area of a circle is $A = \pi * r^2$ where *A* is the area of a circle and *r* is its radius.

- 1. What is the radius of the circle in the Math Message on journal page 373? _____
- 2. Use the formula above to calculate the area of that circle.
- 3. Is the area you found by counting square centimeters more or less than the

area you found by using the formula?

How much more or less? _____

- 4. Use the formula to find the areas of the circles you traced on *Math Masters*, page 2.
- 5. Which do you think is a more accurate way to find the area of a circle, by counting squares or by measuring the radius and using the formula? Explain.

Fraction and Percent Multiplication

Complete the following.

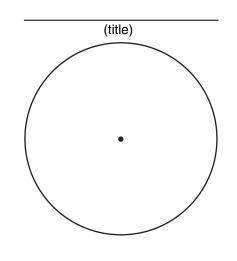
1.	30%	of 50) is _			2.	25%	o of 30	6 is _			3.	5%	of 15	0 is $_{-}$		<u> </u>
4.	75%	of 12	2 is			5.	80%	of 60) is _			6.	50%	6 of 1	30 is		
Find	d the	whole).														
7.	50%	of		is	12.	8.	$\frac{3}{4}$ of			is 21	Ι.	9.	90%	6 of _		is	s 180.
10.	$\frac{5}{6}$ of			is 25		11.	20%	₀ of _		is	s 19.	12.	<u>3</u> 0	of		_ is 2	4.
Mul	tiply.																
13.	$\frac{1}{2} * \frac{1}{2}$	$\frac{3}{4} = -$				14.	2 <u>3</u> :	* <u>3</u> =	:		_	15.	$1\frac{1}{2}$	* 2 <u>1</u>	=		
16.	$\frac{3}{4} *$	5 = _				17.	7 *	$\frac{4}{5} = \frac{1}{2}$				18.	<u>5</u> *	$\frac{1}{5} =$			
																	_
																	_
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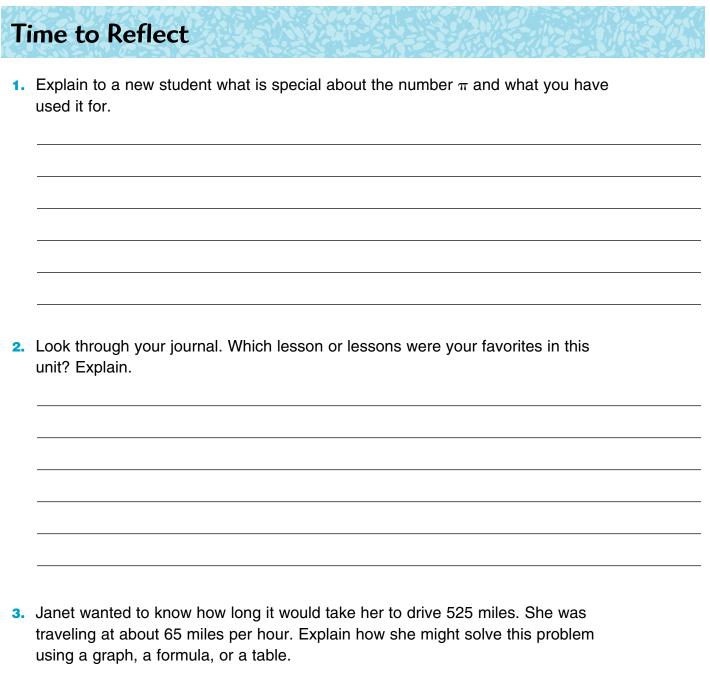
Math	Boxes	10.9
	DUACS	

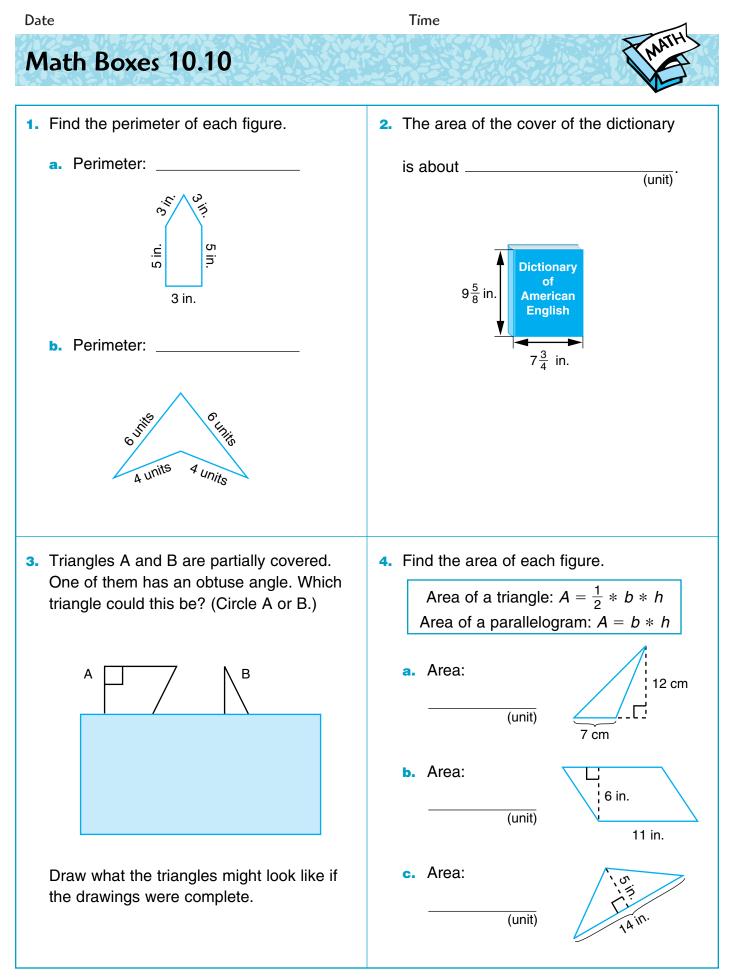
 I am a polygon with exactly 6 angles. What shape am I? 	 2. Add or subtract. a. 24 + (-40) =
Name an object that has my shape somewhere on it.	b. $-7 - (-23) =$ c. $-43 + (-16) =$
	d. $37 - (-37) = $
Do all of my angles have to be the same size? Explain	e10 + 14 =
3. List all of the factors for 205.	4. Add or subtract.
	a. $3\frac{5}{8} - 1\frac{2}{5} = $
	b. $2\frac{6}{7} - \frac{9}{4} = $
	c. = $\frac{37}{5} + 8\frac{3}{2}$
	d. = $\frac{12}{100} + \frac{25}{4}$

5. Ms. Hopheart's class asked 50 people to name their favorite kind of fruit. The results are shown in the table. Complete the table and then make a circle graph of the results.

Kind of Fruit	Number of People	Percent of Total
Apples	20	
Bananas	12	
Grapes	5	
Oranges	8	
Other	5	
Total	50	







Geometric Solids

Each member of your group should cut out one of the patterns from *Math Masters,* pages 150–153. Fold the pattern and glue or tape it together. Then add this model to your group's collection of geometric solids.

1.	Examine your models of geometric solids.				
	a. Wł	/hich solids have all flat surfaces?			
	b. Wł	hich have no flat surfaces?			
	c. Wł	hich have both flat and curved surfaces?			
	line	you cut the label of a cylindrical can in a ne perpendicular to the bottom, and then u attened the label, what would be the shap	unrolled and e of the label?	cut line	
2.	Examine your models of polyhedrons.				
	a. Wł	Which polyhedrons have more faces than vertices?			
	b. Wr	Which polyhedrons have the same number of faces and vertices?			
	c. Wł	Which polyhedrons have fewer faces than vertices?			
3.	Exam	nine your model of a cube.			
		Does the cube have more edges than vertices, the same number of edges as vertices, or fewer edges than vertices?			
	ls t	this true for all polyhedrons?	Explain		
	 ь. Но	. How many edges of the cube meet at each vertex?			
	ls t	this true for all polyhedrons?	Explain		

More Circumference and Area Problems

Circumference and Area of Circles Formulas

Circumference = $\pi * d$ Area = $\pi * r^2$

where d is the diameter and r is the radius of the circle.

Measure the diameter of the circle below to the nearest centimeter. Then use the $\overline{\pi}$ key on your calculator to solve these problems. If your calculator doesn't have a $\overline{\pi}$ key, enter 3.14 each time you need π . Show answers to the nearest tenth.

- 1. The diameter of the circle is _____ cm.
- 2. The radius of the circle is _____ cm.
- 3. The circumference of the circle is _____ cm.
- 4. The area of the circle is _____ cm².

5. Explain the relationship between the diameter and the circumference.

- Use your Geometry Template to draw a circle that has a diameter of 5 centimeters.
 - a. Find the area of your circle.
 - _____ cm²
 - **b.** Find the circumference of your circle.

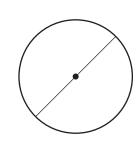
_____ cm

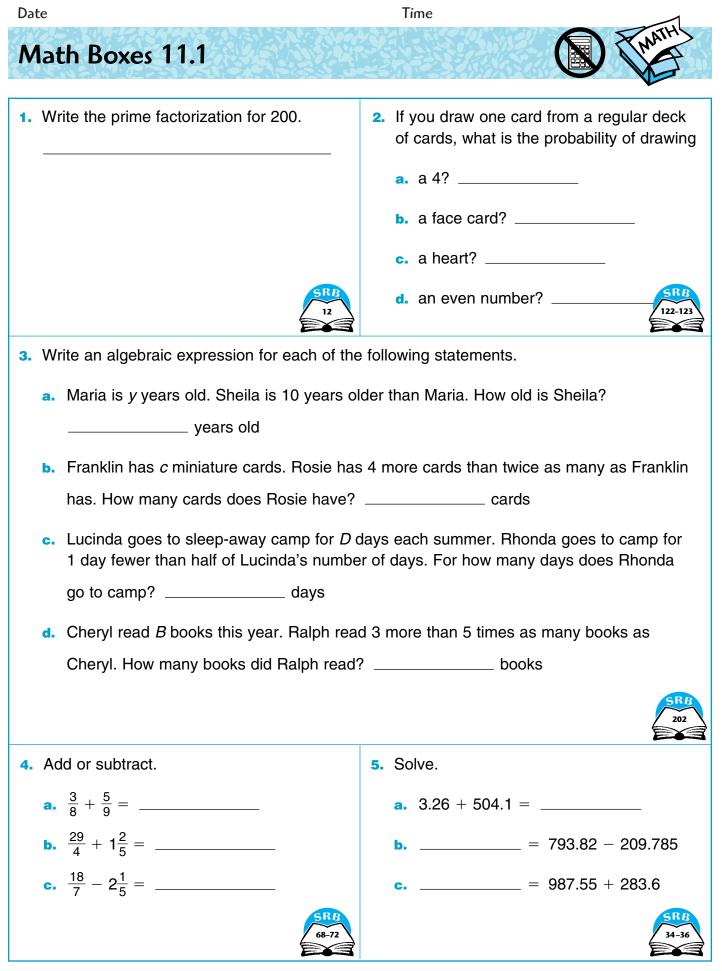
- 7. Use your Geometry Template to draw a circle that has a radius of 1 inch.
 - a. Find the area of your circle.

_____ in.²

b. Find the circumference of your circle.

_____ in.

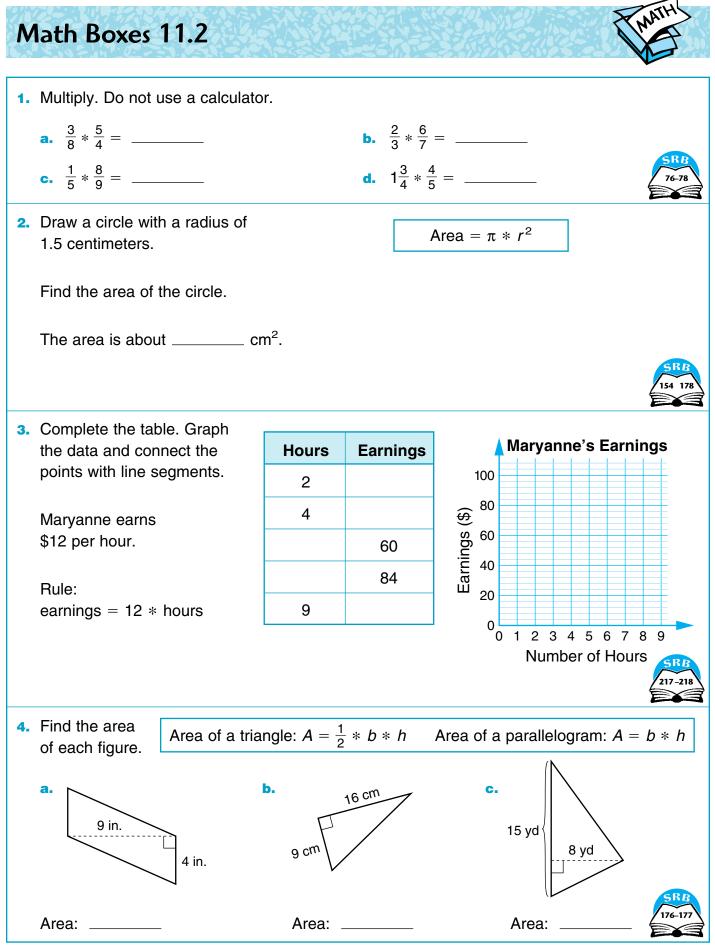




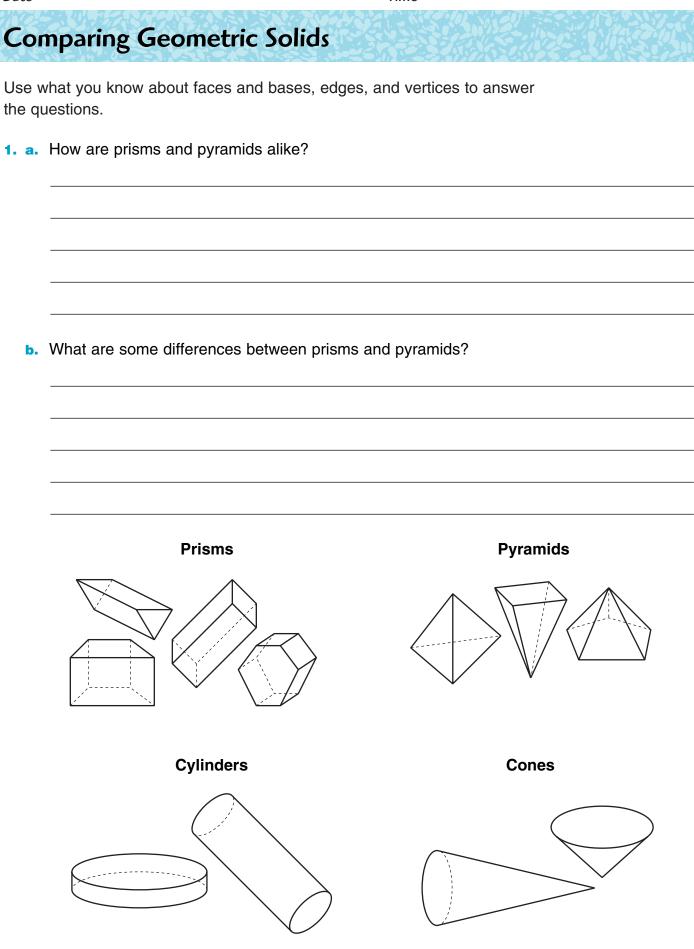
Use with Lesson 11.1.

Date

Time



Use with Lesson 11.2.



С	оп	nparing Geometric Solids (cont.)
2.	а.	How are prisms and cylinders alike?
	b.	What are some differences between prisms and cylinders?
3.	a.	How are pyramids and cones alike?
	b.	What are some differences between pyramids and cones?

Date

Volume of Cylinders

The base of a cylinder is circular. To find the area of the base of a cylinder, use the formula for finding the area of a circle.

Formula for the Area of a Circle

 $A = \pi * r^2$

where A is the area and r is the radius of the circle.

The formula for finding the volume of a cylinder is the same as the formula for finding the volume of a prism.

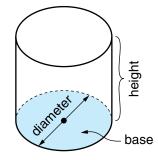
Formula for the Volume of a Cylinder

V = B * h

where V is the volume of the cylinder, B is the area of the base, and h is the height of the cylinder.

Use the two cans you have been given.

- Measure the height of each can, inside the can. Measure the diameter of the base of each can. Record your measurements (to the nearest tenth of a centimeter) in the table below.
- Calculate the radius of the base of each can. Then use the formula to find the volume. Record the results in the table.



3. Record the capacity of each can in the table, in milliliters.

	Height (cm)	Diameter of Base (cm)	Radius of Base (cm)	Volume (cm ³)	Capacity (mL)
Can #1					
Can #2					

4. Measure the liquid capacity of each can. Fill the can with water. Then pour the water into a measuring cup. Keep track of the total amount of water you pour into the measuring cup.

Capacity of Can #1: _____ mL Capacity of Can #2: _____ mL

Use with Lesson 11.3.

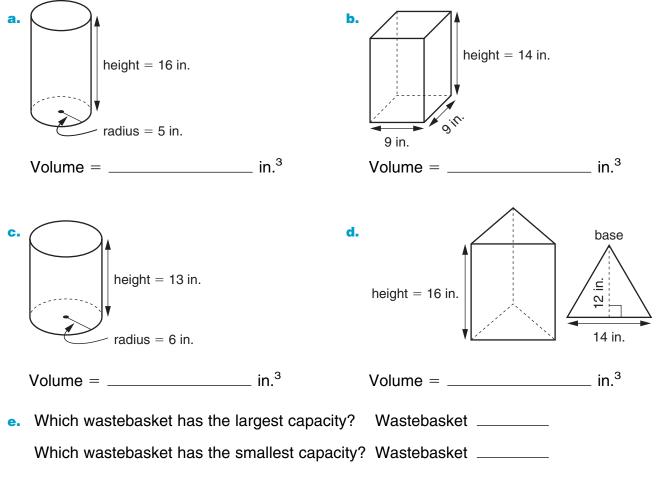
Date Time **Volume of Cylinders and Prisms** 1. Find the volume of each cylinder. a. height = 8 in. Area of base = 10 in.² b. height = 4 cm radius = 2 cm

Reminder: The same formula (V = B * h) may be used to find the volume of a prism and the volume of a cylinder.

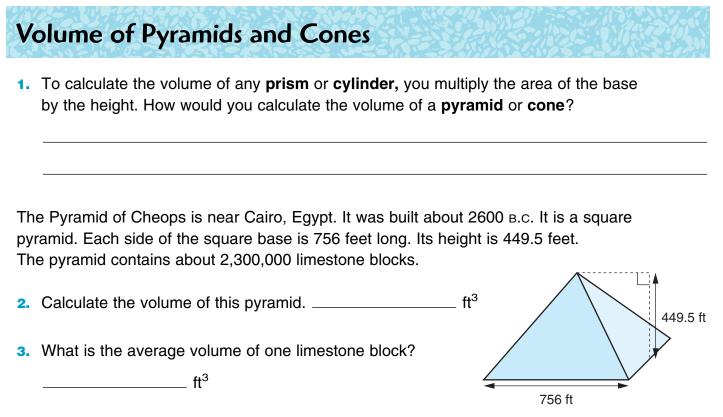
Volume = \dots cm³

2. Find the volume of each wastebasket. Then determine which wastebasket has the largest capacity and which has the smallest.

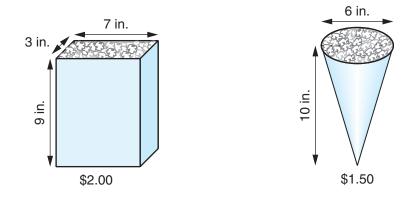
Volume = _____ in.³



Date	Time			
Math Boxes 11.3	The Martin			
Write the prime factorization for 180.	 2. If you roll a regular six-sided die, what is the probability of getting a. a five? b. a prime number? c. an even number? d. a multiple of 3? 			
 3. Theresa is <i>y</i> years old. Write an algebraic expression for the age of each person below a. Nancy is four years older than Theresa. Nancy's age: years b. Frank is twice as old as Theresa. Frank's age: years c. José is ¹/₃ as old as Theresa. José's age: years d. Lucienne is 8 years younger than Theresa. Lucienne's age: years e. If Theresa is 12, who is the oldest person above? How old is that person? 				
4. Add or subtract. a. $4\frac{2}{3}$ b. $2\frac{14}{10}$ c. $8\frac{20}{7}$ $-3\frac{7}{8}$ $+1\frac{8}{9}$ $-6\frac{3}{9}$	 5. Solve. a. 52.6 b. 703.93 c. 826.3 <u>-19.08</u> <u>-251.09</u> <u>+572.91</u> 			



A movie theater sells popcorn in a box for \$2.00. It also sells cones of popcorn for \$1.50 each. The dimensions of the box and the cone are shown below.



- 4. Calculate the volume of the box. _____ in.³
- 5. Calculate the volume of the cone. _____ in.³

Challenge

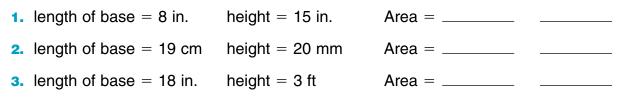
6. Which is the better buy—the box or the cone of popcorn? Explain.



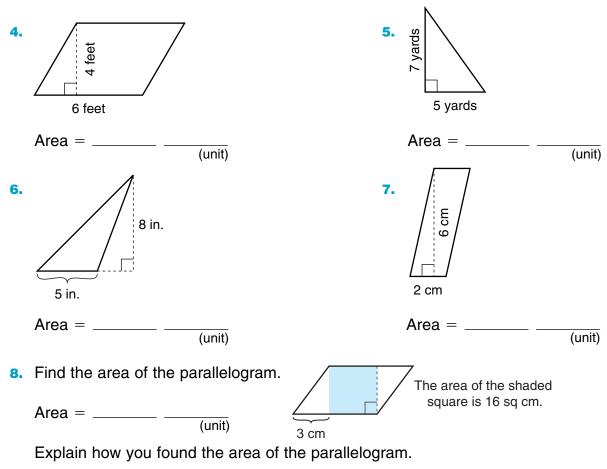


	Area Formulas	
Rectangle:	Parallelogram:	Triangle: $A = \frac{1}{2} * b * h$
A = b * h where A is the area, b is	A = b * h the length of the base, and	E

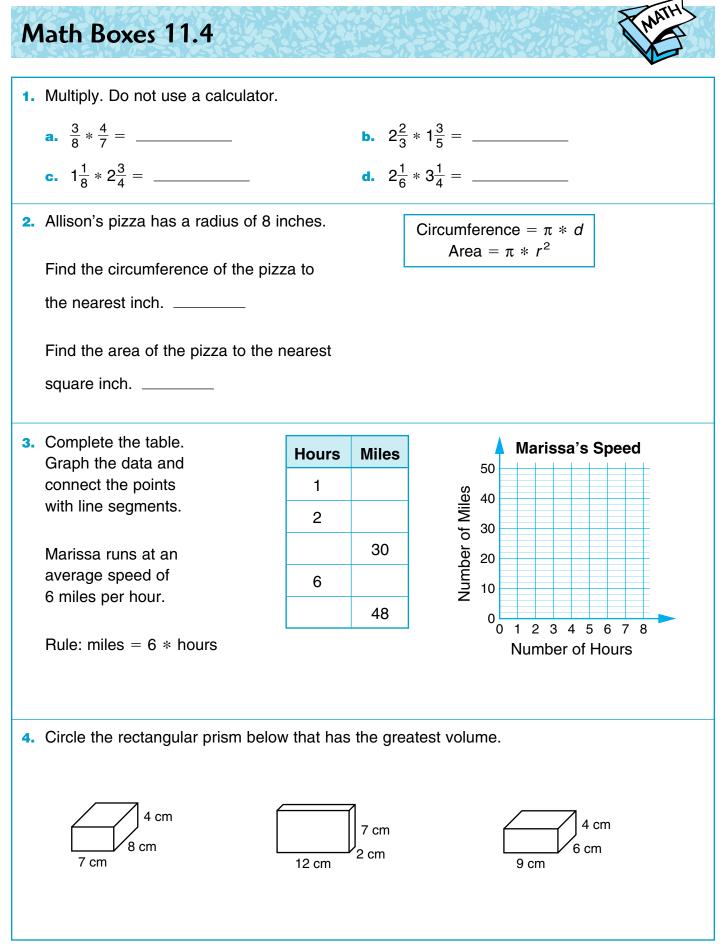
Find the areas of rectangles with the following dimensions. Do not forget the units. You might want to make a sketch of the rectangles on a piece of scratch paper.



Find the area of each of the polygons pictured below.



Date



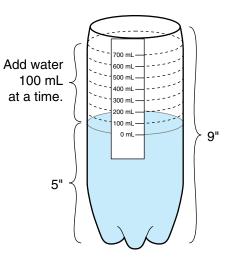
How to Calibrate a Bottle

Materials 2-liter plastic soft-drink bottle with the top cut off

□ can or jar filled with about 2 liters of water

- □ measuring cup □ ruler
- □ scissors □ paper
- 🗅 tape
- 1. Fill the bottle with about 5 inches of water.
- 2. Cut a 1"-by-6" strip of paper. Tape the strip to the outside of the bottle with one end at the bottle top and the other end below the water level.
- Mark the paper strip at the water level. Write "0 mL" next to the mark.
- Pour 100 milliliters of water into a measuring cup. Pour the water into the bottle. Mark the paper strip at the new water level and write "100 mL."
- 5. Pour another 100 milliliters of water into the measuring cup. Pour it into the bottle and mark the new water level "200 mL."
- Repeat, adding 100 milliliters at a time until the bottle is filled to within an inch of the top.
- 7. Pour out the water until the water level in the bottle falls to the 0-mL mark.

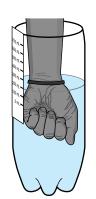
How would you use your calibrated bottle to find the volume of a rock?



Finding Volume by a Displacement Method

- 1. Check that the bottle is filled to the 0-mL level. *Reminder:* 1 mL = 1 cm³ Place several rocks in the bottle. a. What is the new level of the water in the bottle?
 - b. What is the volume of the rocks?
 - c. Does it matter whether the rocks are spread out or stacked? ______
- 2. Your fist has nearly the same volume as your heart. Here is a way to find the approximate volume of your heart. Check that the bottle is filled to the 0-mL level. Place a rubber band around your wrist, just below your wrist bone. Put your fist in the bottle until water reaches the rubber band.
- a. What is the new level of the water in the bottle? b. What is the volume of your fist? This is the approximate volume of your heart.
 - c. Does it matter whether you make a fist or keep your hand open?
- 3. Find the volumes of several other objects in the same way. For example, find the volume of a baseball, a golf ball, an orange, an apple, or a full can of a soft drink. If the object floats in water, use a pencil to force it down. The object must be completely submerged before you read the water level.

Object	Volume of Water Object Displaces (mL)	Volume of Object (cm ³)



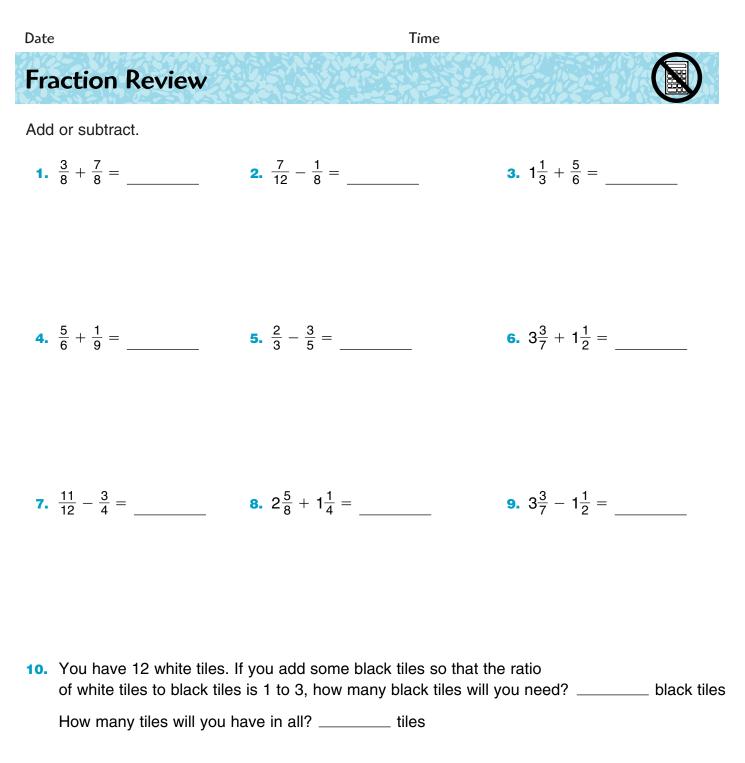
mL

cm³

₋ cm³

393

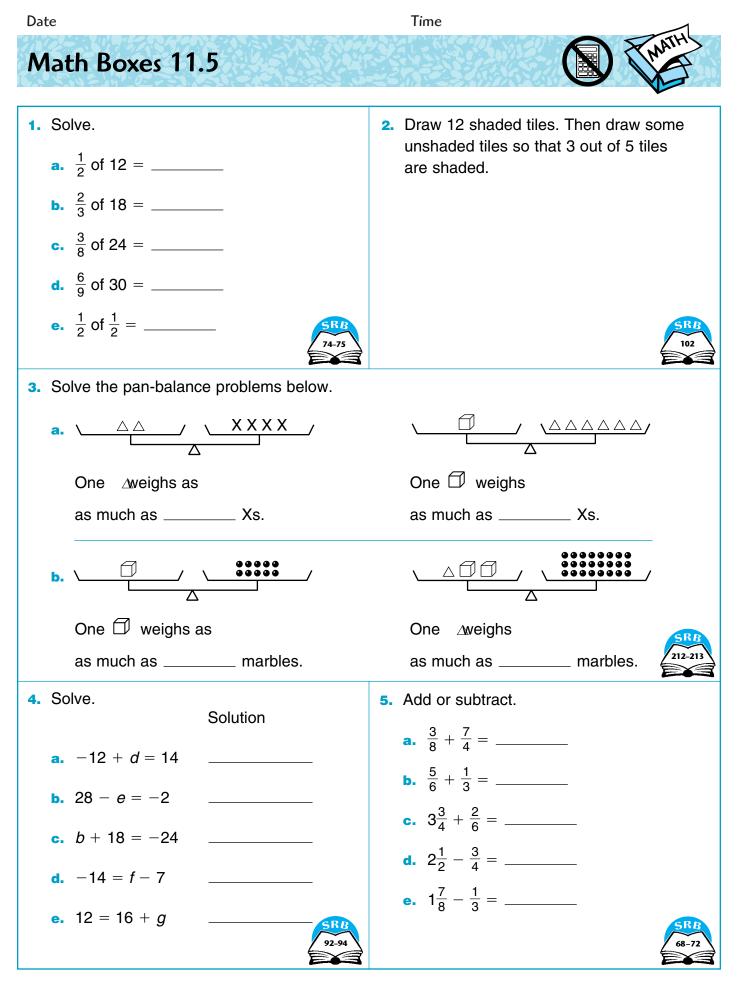
 $_{-} mL$



You have 15 white tiles. If you add some black tiles so that
 3 out of 4 tiles are white, how many black tiles will you need? _____ black tiles

How many tiles will you have in all? ______ tiles

12. You have a total of 24 tiles. Five out of 8 tiles are black.How many black tiles do you have? _____ black tiles

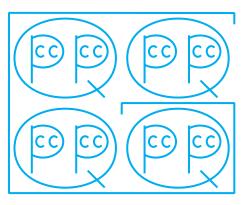


Capacity and Weight

Math Message

- 1 pint = _____ cups
- 1 quart = _____ pints
- 1 half-gallon = _____ quarts
- 1 gallon = _____ quarts

How can the picture above help you remember how many cups are in a pint, how many pints are in a quart, and how many quarts are in a gallon?



1. Round your answer to the nearest ounce.

One cup of dry (uncooked) rice weighs about _____ ounces.

- 2. Use the answer in Problem 1 to complete the following.
 - a. 1 pint of rice weighs about _____ ounces.
 - **b.** 1 quart of rice weighs about _____ ounces.
 - c. 1 gallon of rice weighs about _____ ounces.
 - **d.** 1 gallon of rice weighs about ______ pounds. (1 pound = 16 ounces)
- 3. On average, a family of 4 in Japan eats about 40 pounds of rice a month.

a. That's about how many **pounds** a year? _____

- b. How many gallons a year?
- On average, a family of 4 in the United States eats about
 88 pounds of rice a year. That's about how many gallons a year?
- 5. On average, a family of 4 in Thailand eats about 3 gallons of rice a week.

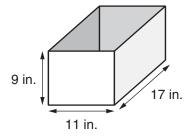
a. That's about how many gallons a year? _____

b. How many pounds a year? _____

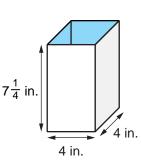
Capacity and Weight (cont.)

6. Find the capacity of the copy-paper carton shown at the right.

_____ in.³



- 7. The container at the right is a $\frac{1}{2}$ -gallon juice container with the top cut off so that $\frac{1}{2}$ gallon of juice exactly fills it.
 - **a.** Find the volume of the $\frac{1}{2}$ -gallon container.
 - What is the volume of a 1-gallon container? ______ in.³



- 8. On average, a family of 4 in Thailand eats about 156 gallons of rice a year. About how many copy-paper cartons will you need to hold this amount of rice? (*Hint:* First calculate how many gallons of rice will fill 1 copy-paper carton.)
 - a. What is the capacity of 1 copy-paper carton?

About _____ gallons

b. How many copy-paper cartons will you need to hold 156 gallons of rice?

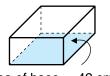
About _____ cartons

Challenge

9. Estimate about how many pounds a copy-paper carton full of rice weighs. Describe what you did to find your estimate.

Da	te		Time		
N	lath Boxes 11.6				MATH
1.	Multiply. Do not use a calculator.				
	a. $1\frac{2}{3} * 2\frac{4}{7} = $	b.	$1\frac{5}{6} * 4\frac{1}{5} =$	=	
	c. = $\frac{18}{3} * \frac{3}{9}$	d.		$_{-}=\frac{7}{8}*\frac{5}{4}$	
	e. = $3\frac{3}{4} * 2\frac{1}{2}$				
2.	What is the diameter of a circle if its ra Radius: <u>/O cm</u> Diameter			Circ	Area = $\pi * r^2$ sumference = $\pi * d$ = $\pi * 2r$
	Find the area and circumference of the Round the area to the nearest square and the circumference to the nearest c	centimete			
	Area: Circumfe	erence:		_	
3.	Fran reads at a rate of 50 pages an hour. Complete the table. Graph the data and connect the points with line segments. Rule: pages = 50 * hours	Hours 1 2 7	Pages 50 150 250	Number of Pages	

4. a. The rectangular prism below has a volume of 126 cubic centimeters.

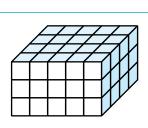


Area of base = 42 cm^2

What is the height

of the prism? _

 b. The prism to the right is made of centimeter cubes.



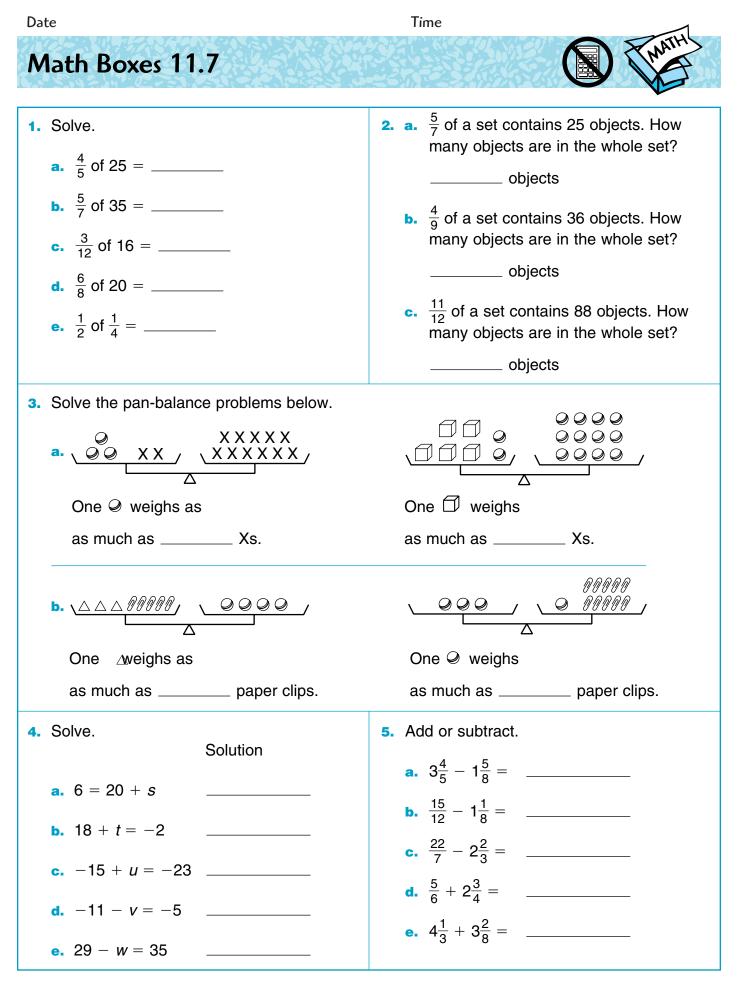
Number of Hours

What is the area

of the base? _____

What is the volume

of the prism? _____



Date

Time

Surface Area The surface area of a box is the sum of the areas of all 6 sides (faces) of the box. 1. Your class will find the dimensions of a cardboard box. a. Fill in the dimensions on the figure below. **b.** Find the area of each side of the box. Then find the total surface area. Area of front = _____ in.² Area of back = $_$ in.² top Area of right side = _____ in.² right side Area of left side = _____ in.² in. front Area of top = _____ in.² in. Area of bottom = _____ in.² ____ in. Total surface area = _____ in.² 2. Think: How would you find the area of all the metal used to manufacture a can? a. How would you find the area of the top or bottom of the can?

b. How would you find the area of the curved surface between the top and bottom of the can?

c. Choose a can. Find the total area of the metal used to manufacture the can. Remember to include a unit for each area.

Area of top = _____

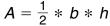
Area of bottom = _____

Area of curved side surface = _____

Total surface area = _____

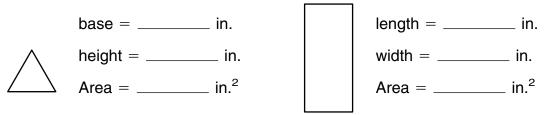


Formula for the Area of a Triangle



where A is the area of the triangle, b is the length of its base, and h is its height.

- **3.** Use your model of a triangular prism.
 - **a.** Find the dimensions of the triangular and rectangular faces. Then find the areas of these faces. Measure lengths to the nearest $\frac{1}{4}$ inch.



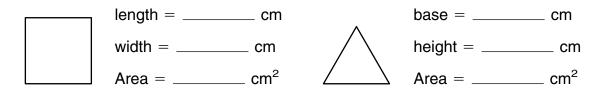
b. Add the areas of the faces to find the total surface area.

Area of 2 triangular bases = _____ in.²

Area of 3 rectangular sides = _____ in.²

Total surface area = _____ in.²

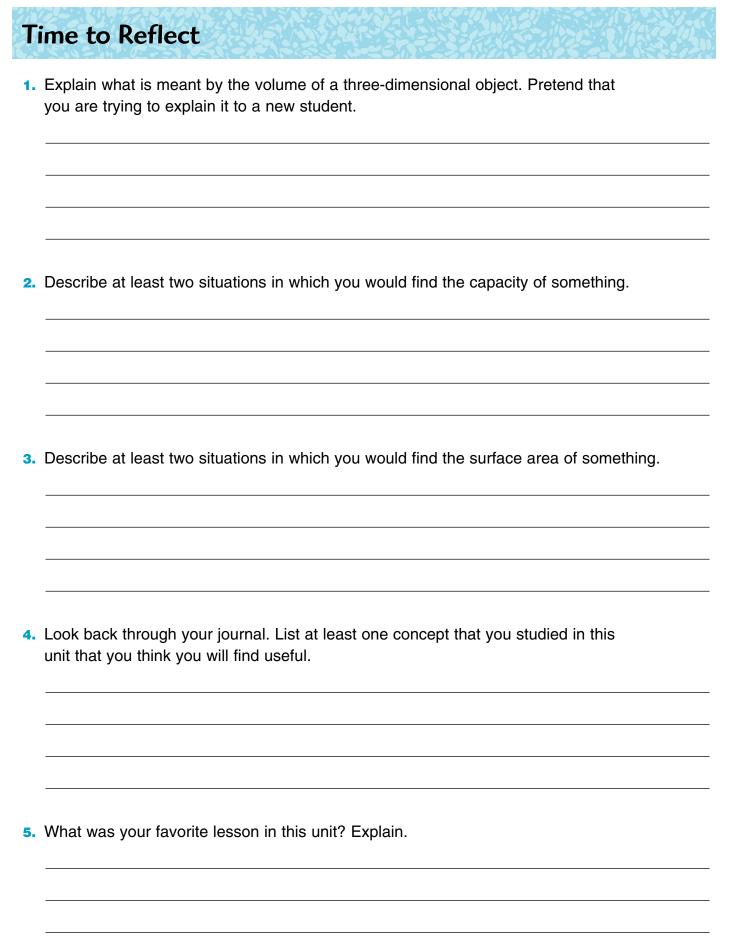
- 4. Use your model of a square pyramid.
 - **a.** Find the dimensions of the square and triangular faces. Then find the areas of these faces. Measure lengths to the nearest tenth of a centimeter.



b. Add the areas of the faces to find the total surface area.

Area of square base = $_ cm^2$ Area of 4 triangular sides = $_ cm^2$

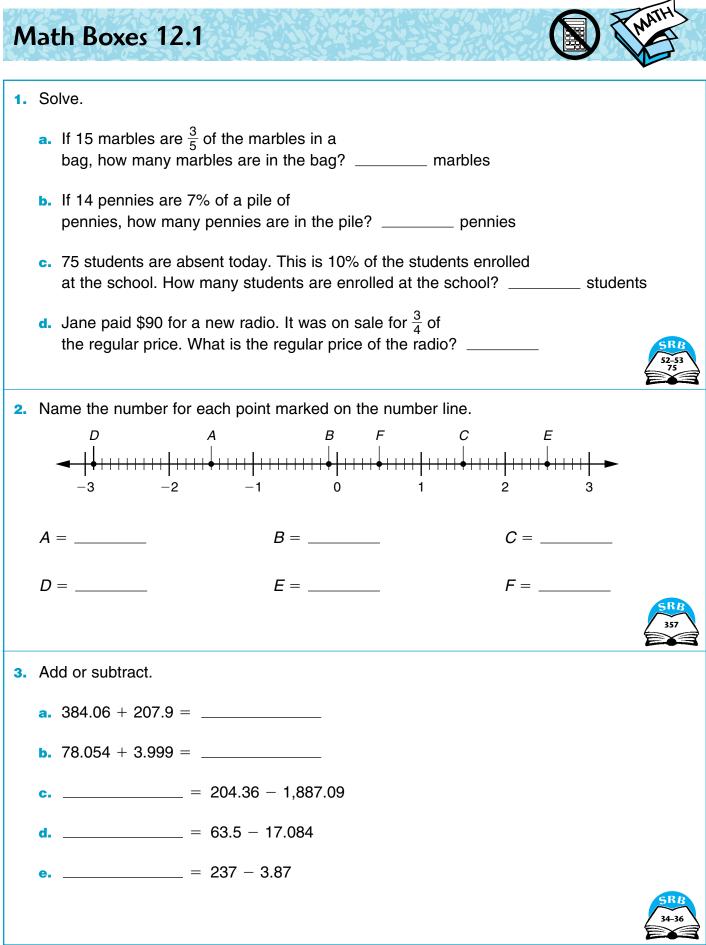
Total surface area = $_$ cm²



Math Boxes 11.8

1.	Write the prime factorization for 175.	2. Solve.				
		a. $\frac{3}{8}$ of 40 =				
		b. $\frac{2}{3}$ of 120 =				
		c. $\frac{4}{5}$ of 60 =				
		d. $\frac{7}{9}$ of 54 =				
		e. $\frac{5}{6}$ of 36 =				
3.	Multiply.	4. Add.				
	a. $\frac{7}{8} * \frac{8}{9} = $	a. $\frac{3}{8} + \frac{2}{5} = $				
	b. = $1\frac{1}{3} * 2\frac{1}{5}$	b. $\frac{5}{6} + \frac{3}{4} = $				
	c. = $4\frac{1}{6} * 3\frac{1}{3}$	c. $1\frac{4}{7} + \frac{2}{3} = $				
	d. = $\frac{25}{6} * \frac{8}{9}$	d. $5\frac{5}{9} + 2\frac{1}{7} = $				
	e. = 5 * $2\frac{5}{7}$	e. $4\frac{1}{5} + 1\frac{7}{8} = $				
5.	Color the spinner so that there is a 25% chail landing on red and a $\frac{1}{3}$ chance of landing on					
	Leave the rest of the spinner white.					
	What is the probability	10/2				
	of landing on white?	9 - • - 3				
	If you spin the spinner 300 times, about how					
	times would you expect the spinner to land o	7 5				
	6					



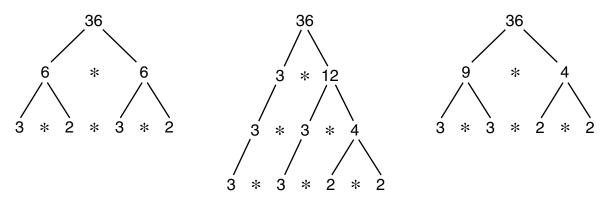




- 1. Write all the pairs of factors whose product is 48. One pair has been done for you. $_{48} = 6 \times 8$
- One way to write 36 as a product of factors is 2 * 18. Another way is 2 * 2 * 9. Write 36 as the product of the longest possible string of factors. Do not include 1 as a factor.

Factor Trees

One way to find all the prime factors of a number is to make a **factor tree**. First, write the number. Then, underneath, write any two factors whose product is that number. Then write factors of each of these factors. Continue until all the factors are prime numbers. Below are three factor trees for 36.



It does not matter which two factors you begin with. You always end with the same prime factors; for 36, they are 2, 2, 3, and 3. The **prime factorization** of 36 is 2 * 2 * 3 * 3.

3. Make a factor tree for each number. Then write the prime factorization.

Factor Trees and Greatest Common Factors

The **greatest common factor** of two whole numbers is the largest number that is a factor of both numbers.

- *Example 1* Find the greatest common factor of 24 and 60.
- Step 1 List all the factors of 24: 1, 2, 3, 4, 6, 8, 12, and 24.
- Step 2 List all the factors of 60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, and 60.
- Step 3 1, 2, 3, 4, 6, and 12 are on both lists. They are common factors.
 12 is the largest number. It is the greatest common factor of 24 and 60.

Another way to find the greatest common factor of two numbers is to use prime factorization.

Example 2 Find the greatest common factor of 24 and 60.

Step 1 Write the prime factorization of each number.

24 = 2 * 2 * 2 * 3 60 = 2 * 2 * 3 * 5

Step 2 Circle pairs of common factors.

Step 3 Multiply one factor in each pair of circled factors.

The greatest common factor of 24 and 60 is 2 * 2 * 3, or 12.

1. Make a factor tree for each number below.

а.	10	b.	75	с.	90
a.	10	D .	75	G.	30

Factor Trees and Greatest Common Factors (cont.)

2.	a. Which prime factors do 10 and 75 have in common?
	b. What is the greatest common factor of 10 and 75?

- 3. a. Which prime factors do 75 and 90 have in common?
 - b. What is the greatest common factor of 75 and 90?
- 4. a. Which prime factors do 10 and 90 have in common?
 - b. What is the greatest common factor of 10 and 90?
- **5.** Use the factor trees in Problem 1 to help you write each fraction below in simplest form. Divide the numerator and denominator by their greatest common factor.

a. $\frac{10}{75} =$	
b. $\frac{75}{90} =$	
c. $\frac{10}{90} =$	

6. What is the greatest common factor of 20 and 25? (*Hint:* Use factor trees to help you.)
 Write the fraction ²⁰/₂₅ in simplest form.

<u>20</u> =

Challenge

7. What is the greatest common factor of 1,260 and 1,350?

(*Hint:* 1,260 = 2 * 2 * 3 * 3 * 5 * 7 and 1,350 = 2 * 3 * 3 * 3 * 5 * 5.)

Factor Trees and Least Common Multiples

The **least common multiple** of two numbers is the smallest number that is a multiple of both numbers.

Example Find the least common multiple of 8 and 12.

Step 1 List the multiples of 8: 8, 16, 24, 32, 40, 48, 56, and so on.

Step 2 List the multiples of 12: 12, 24, 36, 48, 60, and so on.

Step 3 24 and 48 are in both lists. They are common multiples.
24 is the smallest number. It is the least common multiple for 8 and 12.
24 is also the smallest number that can be divided by both 8 and 12.

Another way to find the least common multiple for two numbers is to use prime factorization.

Example Find the least common multiple of 8 and 12.

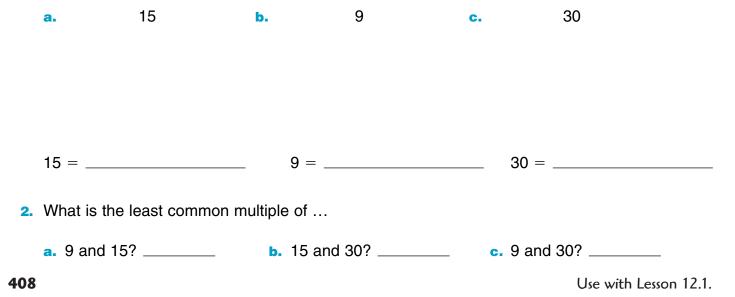
Step 1 Write the prime factorization of each number:

8 = 2 * 2 * 2 12 = 2 * 2 * 3

Step 2 Circle pairs of common factors. Then cross out one factor in each pair as shown below.

$$8 = 2 * 2 * 2
 12 = 2 * 2 * 3$$

- Step 3 Multiply the factors that are not crossed out. The least common multiple of 8 and 12 is 2 * 2 * 2 * 3, or 24.
- 1. Make factor trees and write the prime factorizations for each number.



Da	te Time
R	ate Number Stories
1.	Mica reads about 44 pages in an hour. About how many pages will she read in $2\frac{3}{4}$ hour? pages Explain how you found your answer
	If Mica starts reading a 230-page book at 3:30 P.M., and she reads straight through the book (without stopping), about what time will Mica finish the book? Explain how you found your answer
2.	Tyree and Jake built a tower of centimeter cubes. The bottom floor of the tower is rectangular. It is 5 cubes wide and 10 cubes long. The completed tower is the shape of a rectangular prism. They began building at 2 р.м. They built for about 1 hour. They used approximately 200 cubes every 10 minutes.
	How tall was the final tower?

410

Use with Lesson 12.2.

Probability

When a fair 6-sided die is rolled, each number from 1 to 6 has an equal chance of coming up. The numbers 1, 2, 3, 4, 5, and 6 are equally likely.

Time

The spinner below is divided into 10 equal sections. There is an equal chance of spinning each number from 1 through 10. The numbers 1, 2, 3, ..., 9, 10 are equally likely. This does not mean that if you spin 10 times, each number from 1 to 10 will come up exactly once. A 2 might come up four times, and a 10 might not come up at all. But if you spin many times (say 1,000 times), each number is likely to come up about $\frac{1}{10}$ of the time. The **probability** of landing on 1 is $\frac{1}{10}$. The probability of landing on 2 is also $\frac{1}{10}$, and so on.

Example What is the probability that the spinner at the right will land on an even number?

The spinner will land on an even number if it lands on 2, 4, 6, 8, or 10. Each of these even numbers is likely to come up $\frac{1}{10}$ of the time. The total probability that one of these even numbers will come up is found by adding:

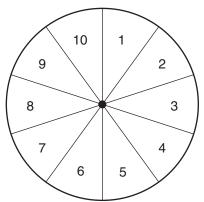
	$\frac{1}{10}$ +	- <u>1</u> 10 -	$+\frac{1}{10}+$	- <u>1</u> 10 -	$+\frac{1}{10} =$	= <u>5</u> 10
Lands on	2	4	6	8	10	

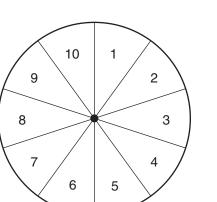
The probability of landing on an even number is $\frac{5}{10}$. Find the probability of each of the following for this spinner.

- The spinner lands on an odd number. 1.
- The spinner lands on a number less than 7. ____ 2.
- The spinner lands on a multiple of 3. 3.
- The spinner lands on a number that is a factor of 12. 4.
- The spinner lands on the greatest common factor of 4 and 6. 5.

The spinner lands on a prime number. _ 6.

The spinner lands on a number that is NOT a prime number.





The Multiplication Counting Principle and Tree Diagrams

Multiplication Counting Principle

Suppose you can make a first choice in m ways, and a second choice in n ways. Then there are m * n ways of making the first choice followed by the second choice. Three or more choices can be counted in the same way, by multiplying.

A school cafeteria offers these choices for lunch:

Main Course: chili or hamburger

Drink: milk or juice

Dessert: apple or cake

1. a. How many different ways can a student choose one main course, one drink, and one dessert? Use the Multiplication Counting Principle.

	*		*			
	(ways to choose a main course)	(ways to choose a drink)	(ways to choose a dessert)			
b.		select foods for lunch:				
. Dra	aw a tree diagram to show all possible ways to select foods for lunch.					
Ма	in Course:					
Dri	nk:					
De	ssert:		· · · ·			
. а.	Do you think that all of the ways to select foods for lunch are equally likely?					
b.	Explain your answer.					

Tree Diagrams and Probability

Sam has 3 clean shirts (red, blue, and yellow) and 2 clean pairs of pants (tan and black). He grabs a shirt and a pair of pants without looking.

1. Complete the tree diagram to show all possible ways that Sam can grab a shirt and a pair of pants.

Shirts:	 	

Pants: _____ ____ ____

2. List all possible combinations of shirts and pants. One has been done for you.

3. How many different combinations of shirts and pants are there? _____ combinations

4. Are all the shirt-pants combinations equally likely?

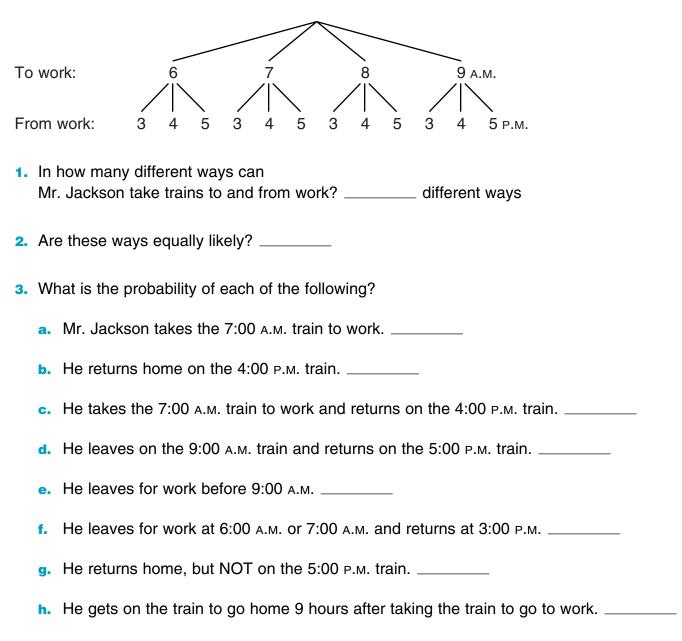
- 5. What is the probability that Sam will grab the following?
 - a. the blue shirt _____
 - b. the blue shirt and the black pants _____
 - c. the tan pants _____
 - d. a shirt that is NOT yellow _____
 - e. the tan pants and a shirt that is NOT yellow _____

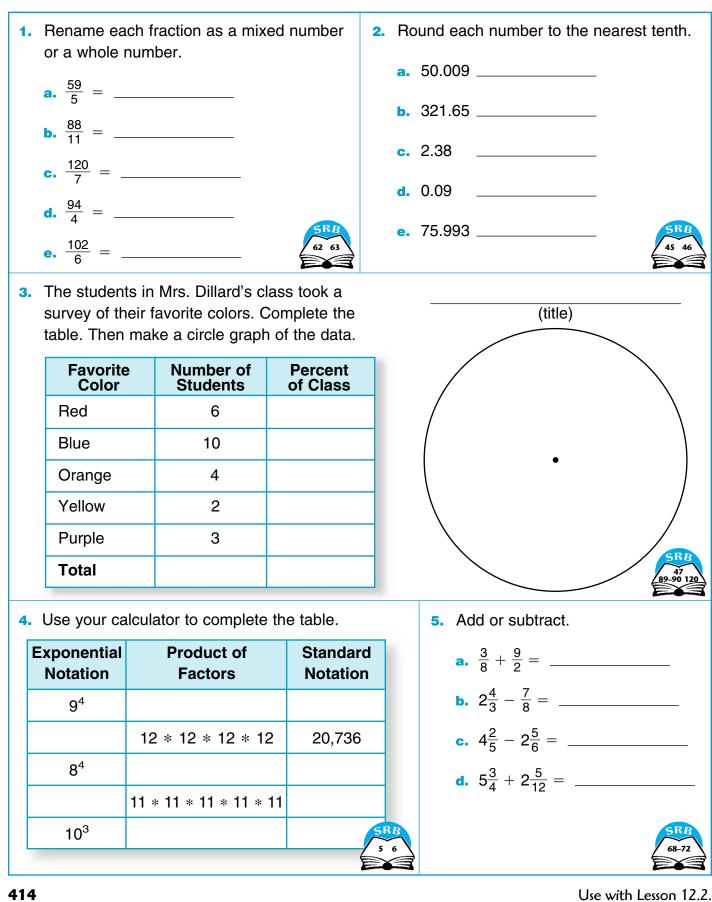
Tree Diagrams and Probability (cont.)

Mr. Jackson travels to and from work by train. Trains to work leave at 6:00, 7:00, 8:00, and 9:00 A.M. Trains from work leave at 3:00, 4:00, and 5:00 P.M.

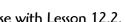
Mr. Jackson is equally likely to select any 1 of the 4 morning trains to go to work.

He is equally likely to select any of the 3 afternoon trains to go home from work.





Math Boxes 12.2



Ratios

Time

Math Message

Ratios can be expressed in many ways. All of the following are statements of ratios:

- It is estimated that by the year 2020 there will be *5 times* as many people 100 years old or older than there were in 1990.
- Elementary school students make up about 14% of the U.S. population.
- On an average evening, about $\frac{1}{3}$ of the U.S. population watches TV.
- The chances of winning a lottery can be less than 1 in 1 million.
- A common scale for dollhouses is 1 inch to 12 inches.

A **ratio** uses division to compare two counts or measures having the same unit. Ratios can be stated or written in a variety of ways. Sometimes a ratio is easier to understand or will make more sense if it is rewritten in another form.

Example In a group of ten students, eight students are right-handed and two are left-handed. The ratio of left-handed students to all students can be expressed in the following ways:

- With words: Two out of the ten students are left-handed.
 - Two in ten students are left-handed.
 - The ratio of left-handed students to all students is two to ten.
- With a fraction: $\frac{2}{10}$, or $\frac{1}{5}$ of the students are left-handed.
- With a percent: 20% of the students are left-handed.
- With a colon between the two numbers being compared: The ratio of left-handed students to all students is 2:10 ("two to ten").

Writing Ratios

Express the ratio of right-handed students to all students in the example above.

- 1. With words: ______ students are right-handed.
- 2. With a fraction: ______ of the students are right-handed.
- 3. With a percent: ______ of the students are right-handed.
- 4. With a colon: The ratio of right-handed students to all students is _____

Using Ratios to Examine a Trend						
1.	а.	According to the table on page 314 of the <i>Student Reference Book,</i> has the ratio of farmers to all working people increased or decreased since 1900?				
	b.	Why do you think this has happened?				
2.	а.	Has the ratio of engineers to all working people increased or decreased since 1900?				
	b.	Why do you think this has happened?				
3.	B. a. How has the ratio of clergy to all working people changed since 1900?					
	b.	Why do you think this has happened?				
Cł	Challenge					
4.	 About how many farmers were there 					
		in 1900?				
5. About how many photographers were there						
	a. in 1900?					
	b.	in 2000?				

10 Times

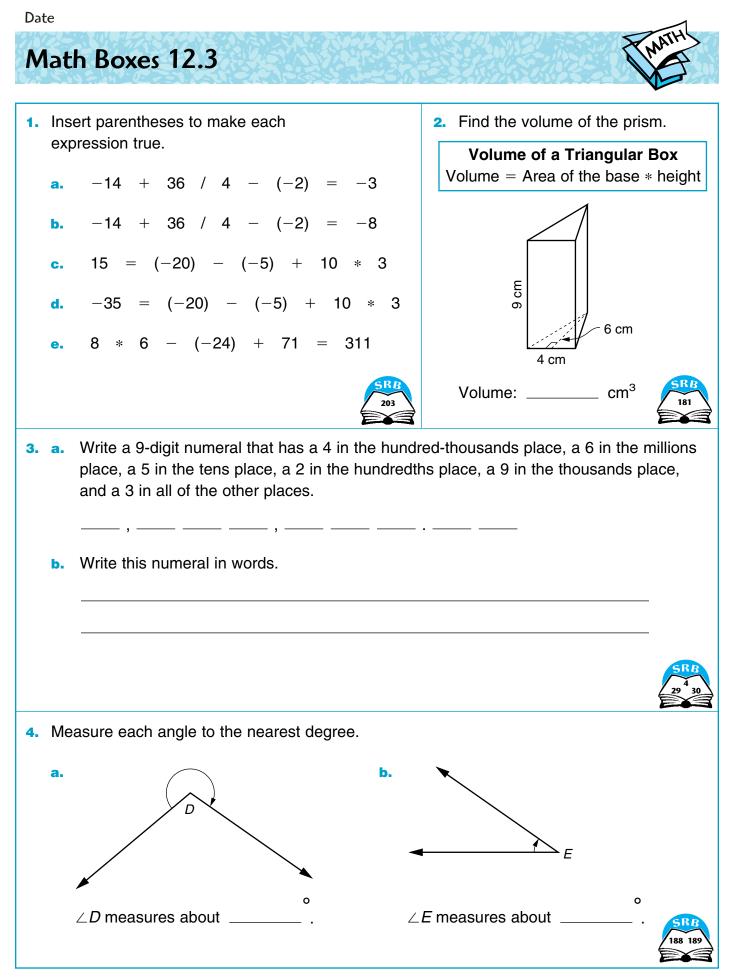
Have you ever heard or used expressions such as "10 times more," "10 times as many," "10 times less," or " $\frac{1}{10}$ as many"? These are **ratio comparisons.** Be sure to use expressions like these with caution. Increasing or reducing something by a factor of 10 makes a big difference!

Scientists call a difference of 10 times a **magnitude**, and they believe that the world as we know it changes drastically when something is increased or decreased by a magnitude.

Example A person can jog about 5 miles per hour. A car can travel 10 times faster than that, or 50 miles per hour. A plane can travel 10 times faster than that, or 500 miles per hour. Each magnitude increase in travel speed has had a great effect on our lives.

Complete the following table. Then add two of your own events or items to the table.

Event or Item	Current Measure or Count	10 Times More	10 Times Less (1 10 as much)
Length of Math Class			
Number of Students in Math Class			
Length of Your Stride			



Use with Lesson 12.3.

Comparing Parts to Wholes

A **ratio** is a comparison. Some ratios compare part of a collection of things to the total number of things in the collection. For example, the statement "1 out of 6 students in the class is absent" compares the number of students absent to the total number of students in the class. Another way to express this ratio is to say, "For every 6 students enrolled in the class, 1 student is absent" or " $\frac{1}{6}$ of the students in the class are absent."

If you know the total number of students in the class, you can use this ratio to find the number of students who are absent. For example, if there are 12 students in the class, then 2 of the students are absent. If there are 18 students in the class, then 3 students are absent.

If you know the number of students who are absent, you can also use this ratio to find the total number of students in the class. For example, if 5 students are absent, there must be a total of 30 students in the class.

Solve the following ratio problems. Use the square tiles you cut out from *Math Journal 2,* Activity Sheet 8 to help you.

1.	Place 28 tiles on your desk so that 1 out	of 4 tiles is white and the rest are shaded.
	How many tiles are white?	How many tiles are shaded?

2. Place 30 tiles on your desk so that 4 out of 5 tiles are white and the rest are shaded.

How many tiles are white? _____ How many tiles are shaded? _____

- Place 7 white tiles on your desk. Add some tiles so that 1 out of 3 tiles is white and the rest are shaded. How many tiles are there in all?
- Place 25 white tiles on your desk. Add some tiles so that 5 out of 8 tiles are white and the rest are shaded. How many tiles are there in all?
- 5. Take 32 tiles. If 6 out of 8 are white, how many are white? _____
- 6. Take 15 tiles. If 6 out of 9 are white, how many are white? _____
- 7. Place 24 tiles on your desk so that 8 are white and the rest are shaded.

One out of _____ tiles is white.

8. Place 18 tiles on your desk so that 12 are white and the rest are shaded.

_____ out of 3 tiles are white.

-71

Time

	atio Number Stories
Je	se your tiles to model and solve the number stories below.
1.	It rained 2 out of 5 days in the month of April. On how many days did it rain that
	month?
2.	For every 4 times John was at bat, he got 1 hit. If he got 7 hits, how many times
	did he bat?
}_	There are 20 students in Mrs. Kahlid's fifth-grade class. Two out of 8 students have no brothers or sisters. How many students have no brothers or sisters?
ı.	Rema eats 2 eggs twice a week. How many eggs will she eat in the month of
	February?
	How many weeks will it take her to eat 32 eggs?
5.	David took a survey of people's favorite flavors of ice cream. Of the people he surveyed, 2 out of 5 said that they like fudge swirl best, 1 out of 8 chose vanilla, 3 out of 10 chose maple walnut, and the rest chose another flavor.
	a. If 16 people said that fudge swirl is their favorite flavor, how many people took
	part in David's survey?
	b. If 80 people participated in David's survey, how many preferred a flavor that is
	not fudge swirl, vanilla, or maple walnut?
6.	Make up your own ratio number story. Ask your partner to solve it.

Date Time Math Boxes 12.4 1. Martin missed $\frac{1}{8}$ of the 24 shots he took in a basketball game against the Rams. a. What fraction of the shots did he make? b. How many shots did he miss? ______ shots c. How many shots did he make? _____ shots d. What percent of his shots did he make? **2. a.** Mark and label -1.7, 0.8, -1.3, and 1.9 on the number line. ····· -2 -1 2 b. What number is 1 less than -1.7? _____ c. What number is 1 more than 1.9? 3. Add or subtract. 703.03 b. 243.84 d. a. C. 438.29 278.6 665.4 176.56 + 105.003 - 89.45

More Ratio Number Stories

You can solve ratio number stories by first writing a number model for the story.

Example Sidney missed 2 out of 9 problems on the math test. There were 36 problems on the test. How many problems did he miss? 1. Write a number model: $\binom{\text{(missed)}}{\text{(total)}} \frac{2}{9} = \frac{1}{36}$
2. Find the missing number.
Think: 9 times what number equals 36 ? 9 * 4 = 36
Multiply the numerator, 2, by this number: $2 * 4 = 8$ (missed) $\frac{2 * 4}{9 * 4} = \frac{8}{36}$ (total)
3. Answer: Sidney missed 8 out of 36 problems.

Write a number model for each problem. Then solve the problem.

1. Of the 42 animals in the Children's Zoo, 3 out of 7 are mammals. How many mammals are there in the Children's Zoo?

Number model: _____ Answer: _____ (unit)

2. Five out of 8 students at Kenwood School play an instrument. There are 224 students at the school. How many students play an instrument?

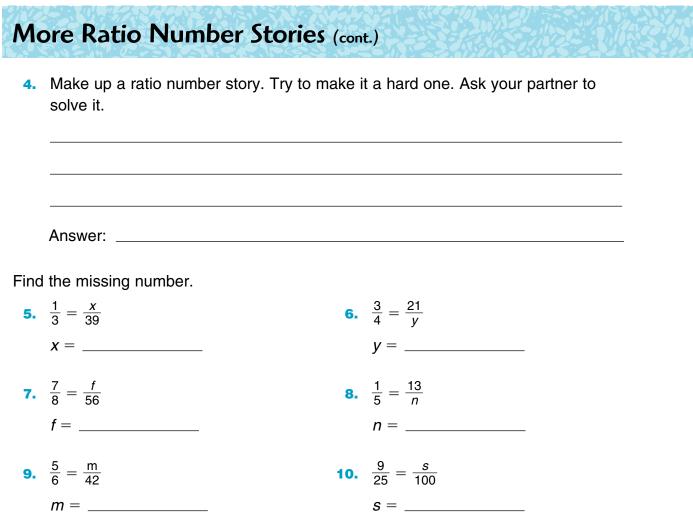
Number model: _____ Answer: _____(unit)

3. Mr. Lopez sells subscriptions to a magazine. Each subscription costs \$18. For each subscription he sells, he earns \$8. One week, he sold \$198 worth of subscriptions. How much did he earn?

Number model: _____ Answer: \$ _____

Date

Time



Challenge

11. There are 48 students in the fifth grade at Robert's school. Three out of 8 fifth graders read two books last month. One out of 3 students read just one book. The rest of the students read no books at all.

How many books	in all did the fifth graders read last month?	
,		(unit)

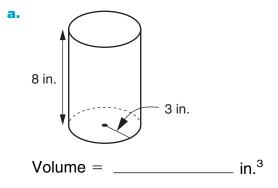
Explain what you did to find the answer.

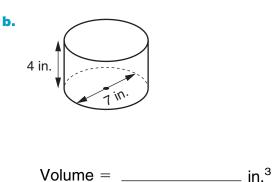
Volume Review

Date

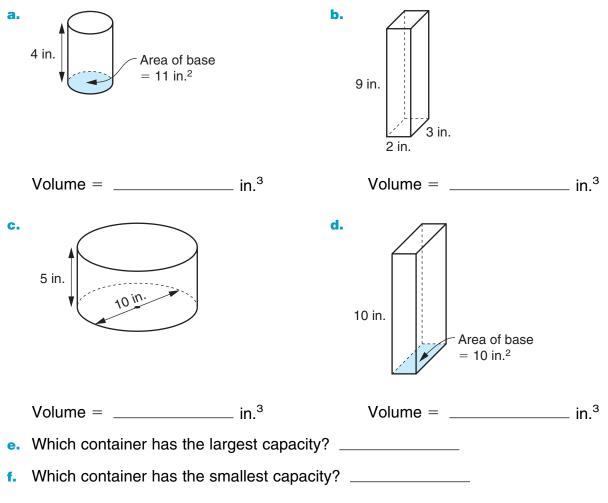
Area of rectangle: A = b * hVolume of prism or cylinder: V = B * h Area of circle: $A = \pi * r^2$ Circumference of circle: $C = 2 * \pi * r$

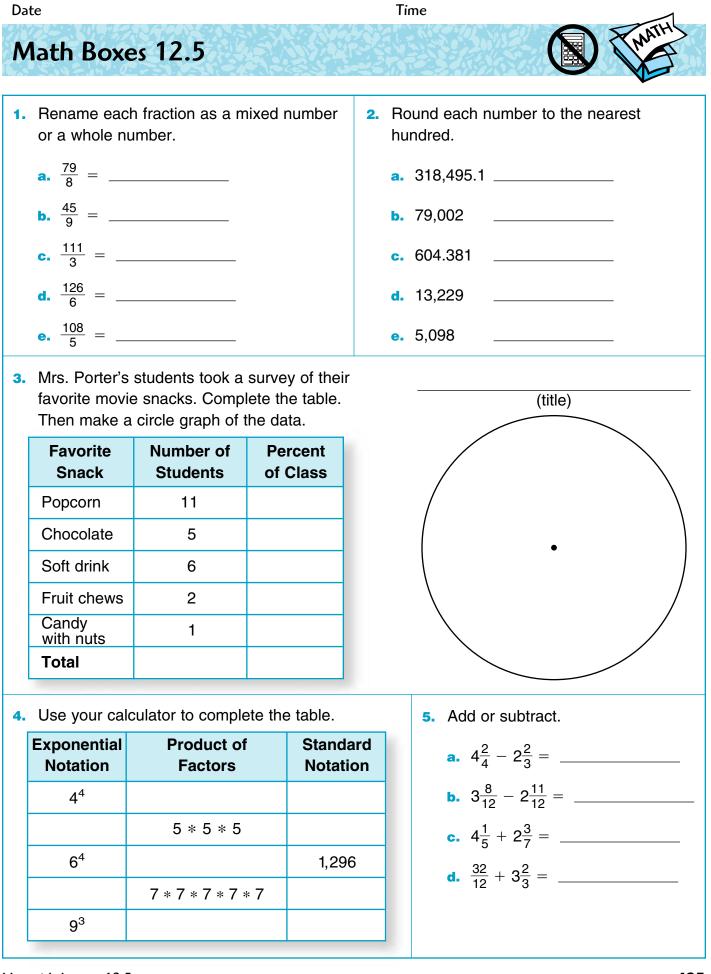
1. Find the volume of each cylinder.





2. Four food containers are pictured below. Find the volume of each. Determine which container has the largest capacity and which has the smallest capacity.

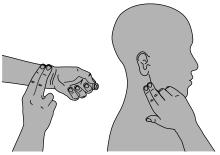




Use with Lesson 12.5.

The Heart

The heart is an organ in your body that pumps blood through your blood vessels. **Heart rate** is the rate at which your heart pumps blood. It is usually expressed as the number of heartbeats per minute. With each heartbeat, the arteries stretch and then go back to their original size. This throbbing of the arteries is called the **pulse.** The **pulse rate** is the same as the heart rate.



You can feel your pulse along your wrist, near the bone, and below the thumb. You can also feel it in your neck: Run your index and middle fingers from your ear, past the curve of your jaw, and press them into the soft part of your neck just below your jaw.

My Heart Rate

Feel your pulse and count the number of heartbeats in 15 seconds. Your partner can time you with a watch or the classroom clock. Do this several times, until you are sure that your count is accurate.

About how many times does your heart beat in 15 seconds?

2. At this rate, about how many times would it beat in 1 minute?

in 1 hour? _____

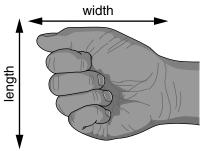
in 1 day? _____

in 1 year? _____

3. Your fist and your heart are about the same size. Measure your fist with your ruler. Record the results.

My heart is about _____ inches wide

and ______ inches long.



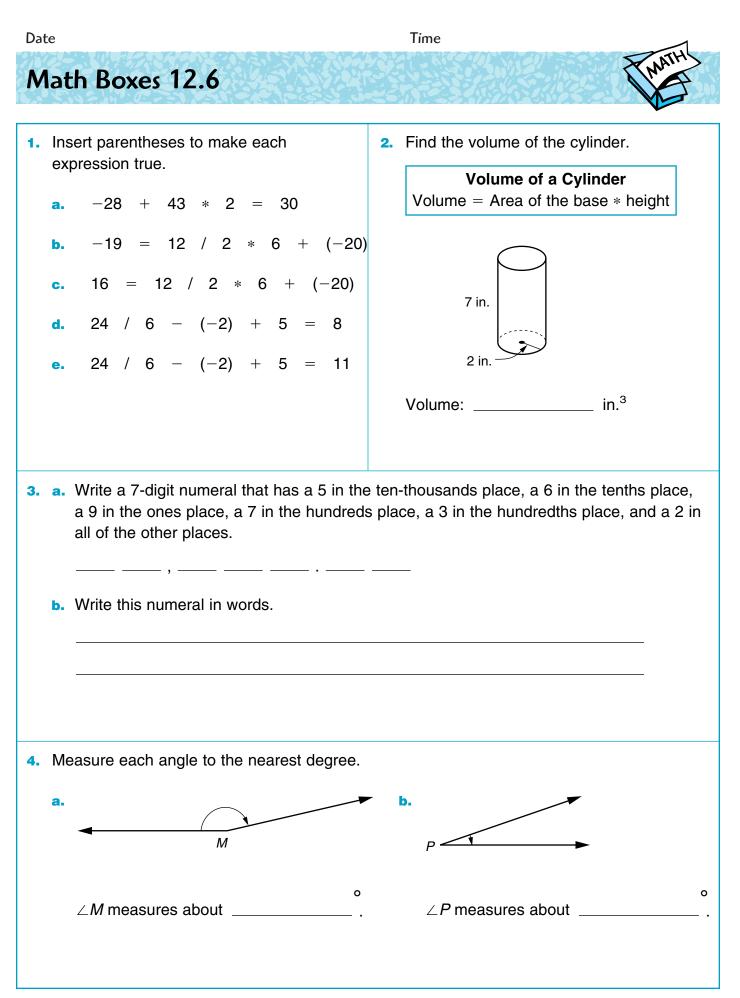
4. A person's heart weighs about 1 ounce per 12 pounds of body weight.

Circle how much your heart weighs.

Less than 15 ounces

About 15 ounces

More than 15 ounces



Exercise and Your Heart

Exercise increases the rate at which a person's heart beats. Very strenuous exercise can double the heart rate.

Work with a partner to find out how exercise affects your heart rate.

- Sit quietly for a minute. Then have your partner time you for 15 seconds while you take your pulse. Record the number of heartbeats in the first row of the table at the right.
- Step up onto and down from a chair 5 times without stopping. As soon as you finish, take your pulse for 15 seconds while your partner times you. Record the number of heartbeats in the second row of the table.
- Sit quietly. While you are resting, your partner can do 5 step-ups, and you can time your partner.
- 4. When your pulse is almost back to normal, step up onto and down from the chair 10 times. Record the number of heartbeats in 15 seconds in the third row of the table. Then rest while your partner does 10 step-ups.
- 5. Repeat for 15, 20, and 25 step-ups.
- 6. Why is it important that all students step up at the same rate?

Step-ups	Heartbeats per 15 Seconds
0	
5	
10	
15	
20	
25	

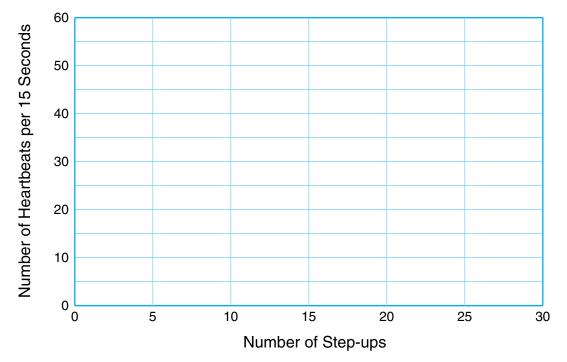


Have a Heart

Giraffes do! Their hearts weigh up to 25 pounds and are up to 2 feet across. A giraffe's heart has to work hard to move blood up that neck, which can be 10 to 12 feet long. The average giraffe's blood pressure is three times that of a human's.

Source: Beyond Belief!

My Heart-Rate Profile



1. Make a line graph of the data in your table on journal page 428.

2. Make a prediction: What will your heart rate be if you do 30 step-ups?

About ______ heartbeats in 15 seconds

3. When you exercise, you must be careful not to put too much stress on your heart. Exercise experts often recommend a "target" heart rate to reach during exercise. The target heart rate varies, depending on a person's age and health, but the following rule is sometimes used.

Target heart rate during exercise:

Subtract your age from 220. Multiply the result by 2. Then divide by 3.

The result is the target number of heartbeats per minute.

a. According to this rule, what is your target heart rate during exercise?

About _____ heartbeats per minute

b. That's about how many heartbeats in 15 seconds?

About _____ heartbeats

My Class's Heart-Rate Profile

1. Complete the table.

Class Landmarks: Number of Heartbeats per 15 Seconds				
Number of Step-ups	Maximum	Minimum	Range	Median
0				
5				
10				
15				
20				
25				

- 2. Make a line graph of the medians on the grid on journal page 429. Use a coloring pencil or crayon. Label this line "Class Profile." Label the other line "My Own Profile."
- 3. Compare your personal profile to the class profile.



Miles of Blood

There are about 5 million red blood cells, and between 5 thousand and 10 thousand white blood cells, in 1 milliliter of blood from an average man. If all of one man's blood cells were lined up side by side, they would wrap around Earth about seven times.

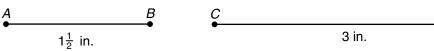
Source: The Odd Book of Data

Date				Time		
Ma	th Boxes	12.7				MATH
1. M	ark and label	each point on	the ruler below	V.		
A:	: 4 <mark>1</mark> in.	<i>B:</i> ³ / ₁₆ in.	<i>C:</i> 2 ⁷ / ₈ in.	<i>D:</i> 1 ¹ / ₂ in.	<i>E:</i> 3 ³ / ₈ in.	
	1 NCHES	2	3	4	5	
	ultiply or divid				/	
a.			99 <u>37</u>	c. 9)243	d. 84)856	
						SR 19 20 22-24
3. So	$\bigcirc \bigcirc$		100	666		
a.					/ <u>(1118X</u> 	/
	One orange			One cube weig		
_	as much as		Xs.	as much as	X:	S
b.			00000 00000 j	<u>6X 0000</u>	00000000 / <u>\4X 00000000</u> 	
	One triangle			One paper clip		
	as much as		X.	as much as $_$	X.	SRB 212 213

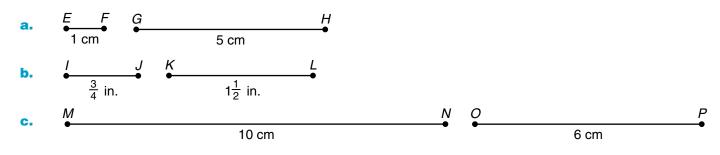
Date

Review of Ratios

1. What is the ratio of the length of line segment AB to the length of line segment CD?



2. Circle the pair of line segments whose lengths have the same ratio as \overline{AB} to \overline{CD} in Problem 1.



3. There are 13 boys and 15 girls in a group. What fractional part of the group

is boys?

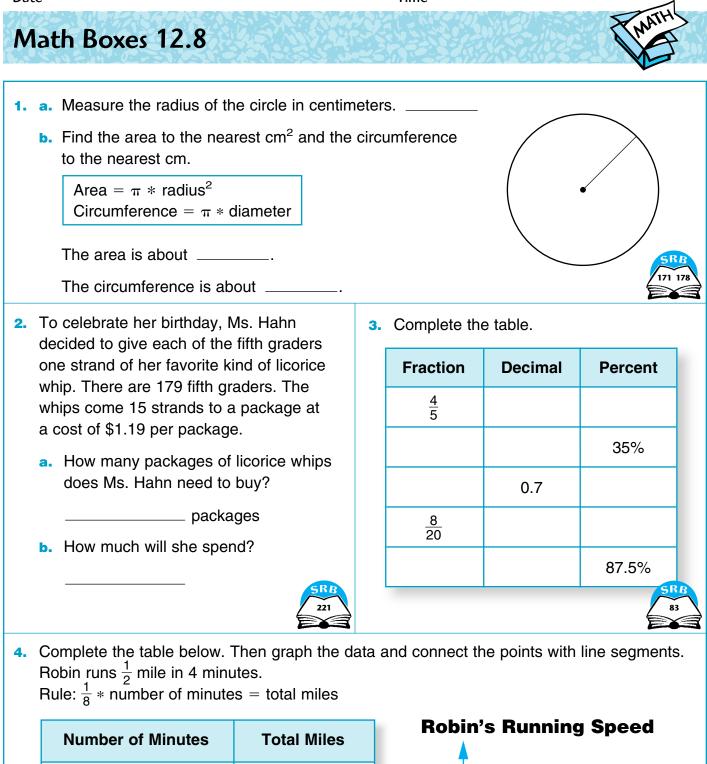
4. Problem 3 was given to groups of 13-year-olds, 17-year-olds, and adults. The answers and the percent of each group that gave those particular answers are shown in the table below.

Answers	13-Year-Olds	17-Year-Olds	Adults
<u>13</u> 28	20%	36%	25%
$\frac{13}{28}$ written as a decimal	0%	0%	1%
13/15 or 0.86	17%	17%	15%
<u>15</u> 28	2%	2%	3%
Other incorrect answers	44%	29%	35%
Don't know	12%	13%	20%
No answer	5%	3%	1%

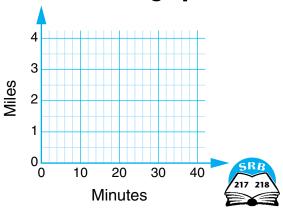
a. What mistake was made by the people who gave the answer $\frac{15}{28}$?

b. What mistake was made by the people who gave the answer $\frac{13}{15}$?

D



Number of Minutes	Total Miles
4	$\frac{1}{2}$
8	
	$3\frac{1}{2}$
32	



The Heart Pump

Your heart is the strongest muscle in your body. It needs to be, because it never rests. Every day of your life, 24 hours a day, your heart pumps blood throughout your body. The blood carries the **nutrients** and **oxygen** your body needs to function.

You breathe oxygen into your lungs. The oxygen passes from your lungs into your bloodstream. As your heart pumps blood throughout your body, the oxygen is deposited in the cells of your body and is replaced by waste products (mainly **carbon dioxide**). The blood carries the carbon dioxide back to your lungs, which get rid of the carbon dioxide as you exhale. The carbon dioxide is replaced by oxygen, and the cycle begins again.

The amount of blood the heart pumps in 1 minute is called the **cardiac output**. To find your cardiac output, you need to know your **heart rate** and the average amount of blood your heart pumps with each heartbeat. Cardiac output is calculated as follows:

Cardiac output = amount of blood pumped per heartbeat * heart rate

On average, the heart of a fifth grader pumps about 1.6 fluid ounces of blood with each heartbeat. If your heart beats about 90 times per minute, then your heart pumps about 1.6 * 90, or 144 fluid ounces of blood per minute. Your cardiac output would be about 144 fluid ounces, or $1\frac{1}{8}$ gallons of blood per minute. That's about 65 gallons of blood per hour. Imagine having to do this much work, around the clock, every day of your life! Can you see why your heart needs to be very strong?

A person's normal heart rate decreases with age. A newborn's heart rate can be as high as 110 to 160 beats per minute. For 10-year-olds, it is around 90 beats per minute; for adults, it is between 70 and 80 beats per minute. It is not unusual for older people's hearts to beat as few as 50 to 65 times per minute.

Because cardiac output depends on a person's heart rate, it is not the same at all times. The more often the heart beats in 1 minute, the more blood is pumped throughout the body.

Exercise helps your heart grow larger and stronger. The larger and stronger your heart is, the more blood it can pump with each heartbeat. A stronger heart needs fewer heartbeats to pump the same amount of blood. This puts less of a strain on the heart.

The H	eart	Pum	D	(cont.)
	C urc			conc.)

Pretend that your heart has been pumping the same amount of blood all of your life so far—about 65 gallons of blood per hour.

1. a. At that rate, about how many gallons of blood would your heart pump per day?

About _____ gallons

- b. About how many gallons per year? About ______ gallons
- At that rate, about how many gallons would it have pumped from the time you were born to your last birthday? About ______ gallons
- 3. Both heart rate and cardiac output increase with exercise. Look at the table on journal page 428. Find the number of heartbeats in 15 seconds when you are at rest and the number of heartbeats after 25 step-ups. Record them below.
 - a. Heartbeats in 15 seconds at rest:
 - b. Heartbeats in 15 seconds after 25 step-ups: _____

Now figure out how many heartbeats in 1 minute.

- c. Heartbeats in 1 minute at rest: _____
- d. Heartbeats in 1 minute after 25 step-ups:
- 4. If your heart pumps about 1.6 fluid ounces of blood per heartbeat, about how much blood does it pump in 1 minute when you are at rest?

About _____ fl oz

5. A gallon is equal to 128 fluid ounces. About how many gallons of blood does your heart pump in 1 minute when you are at rest?

About _____ gallon(s)

6. a. Use your answer to Problem 5 above to find about how many fluid ounces of blood your heart would pump in 1 minute after 25 step-ups.

About _____ fl oz

b. About how many gallons? About ______ gallon(s)

Use with Lesson 12.8.

American Tour: End-of-Year Projects

Work with a partner or in a small group on one or more of the following projects, or think up a project of your own. Each project has four steps.

- Step 1 Plan and Do Research. Use the American Tour section of the *Student Reference Book* and other reference sources such as encyclopedias, almanacs, and the Internet to locate necessary and helpful data. Decide which information to use.
- Step 2 Analyze Data. In order to complete the project, you will need to analyze and possibly transform the data you find in your sources.
- Step 3 Record and Display Your Findings. Write a journal; make charts, graphs, tables, and other displays to record and show what you have found.
- Step 4 Present Your Results. Report your findings to your classmates in clear and interesting ways.

Project 1: "Most" State, "Least" State, My State

Look up a variety of population and environmental statistics in the American Tour and in other sources. Create a display that shows which state has the most or the highest number, which has the least or lowest number, and the number for your state (if it is different). Then write a sentence or two for each comparison that describes how your state compares to the "most" and "least" states.

For example, if you live in Connecticut, you might make a comparison like the one below:

	Population ir	n 2000
Most	California	32,521,000 people
Least	Wyoming	525,000 people
My State	Connecticut	3,284,000 people

My state's population is about $\frac{1}{10}$ the population of California, but about 6 times the population of Wyoming.

This is just one way you could make the comparison. There are many others. Find ways to make interesting and informative comparisons.

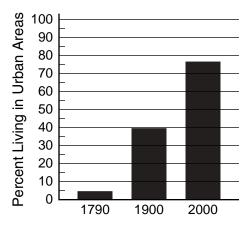
American Tour: End-of-Year Projects (cont.)

Project 2: Then and Now

The American Tour section of the *Student Reference Book* contains information about the United States during your lifetime and approximately 100 and 200 years ago. Use some of this information, as well as information from other sources, to create a series of bar graphs that compare the United States of your lifetime to the United States of approximately 100 and 200 years ago. For each graph, write a newspaper headline that describes an interesting pattern or fact shown by the graph.

For example, one of the bar graphs might compare the percent of the population living in urban areas in 1790, 1900, and 2000. It might look like the graph at the right.

Percent of U.S. Population Living in Urban Areas Increases 15-Fold in 200 Years!



Some hints to keep in mind:

- Sometimes there will be no data for approximately 200 years ago. If this is the case, then compare data for approximately 100 years ago with the present.
- The dates do not have to be the same for each bar graph. Just make sure to note on each graph which years you are comparing.
- Clearly label your graphs. Give them titles, and indicate which counts, measures, or percents you are comparing and the years for which you have data.

Project 3: State Almanac

Use the American Tour and other sources to create a State Almanac of interesting facts and features about your state (or another state). You might include the following information:

- the year your state became a state
- the number of Native Americans who lived in your state in 2000
- the number of times greater your state's population was in 2000 than in 1900

Illustrate the State Almanac with graphs, pictures, and other displays that highlight special features of your state.

Use with Lesson 12.9.

American Tour: End-of-Year Projects (cont.)

Project 4: A Westward Journey

Use the information in the American Tour section of the *Student Reference Book* to help you write a journal that describes a trip across the country in 1840.

Begin the trip at a city on the east coast. From there, travel to St. Louis. Make part of this journey by foot, part by horseback, and part by stagecoach.

From St. Louis, take the trail of your choice west. Assume there is a road from St. Louis to Independence along the Missouri River.

Make approximately half of the journey from St. Louis to a city on the west coast by stagecoach and half by wagon train.

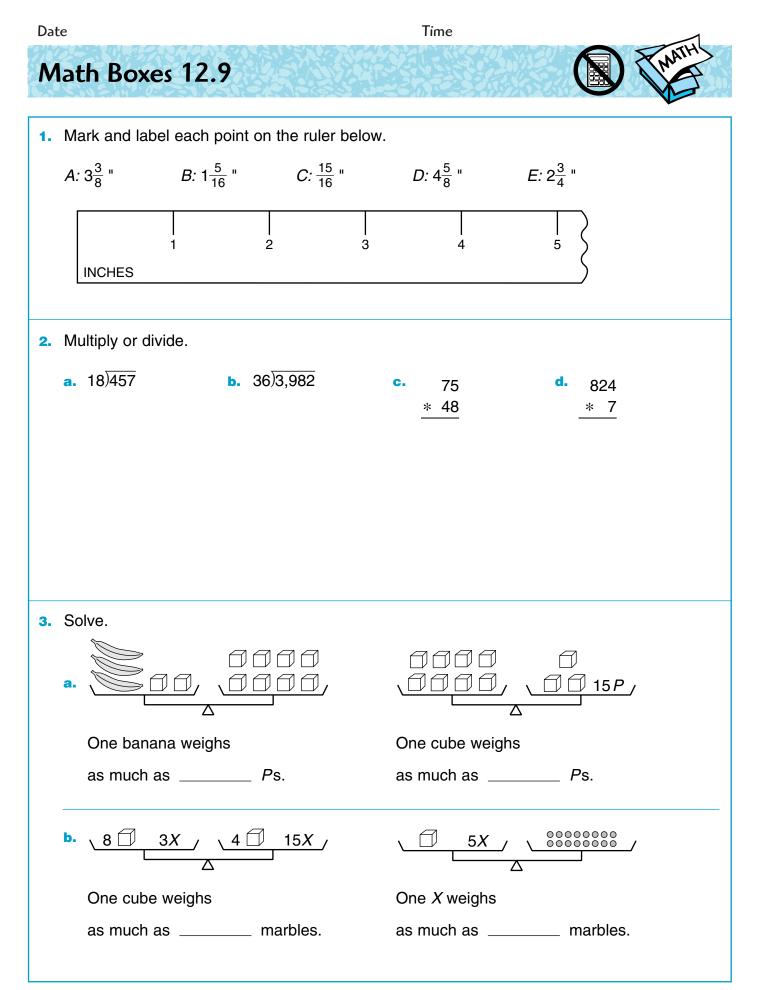
Find the number of days each part of the trip will take and the total traveling time from coast to coast.

You will need to make other decisions. How many hours per day could you travel by the various means of transportation? Do you need to rest along the way? Use your imagination.

For travel between cities in the northeast and St. Louis, you can use the highway map on page 346 as a rough guide to distances between cities. For travel west of St. Louis, use the map and scale on page 312.

You might, for example, begin as follows:

June 1	We departed Boston by stagecoach. Our destination was New York.
June 3	We arrived in New York. The journey from Boston took 2 days. The stagecoach traveled about 12 hours a day, covering around 100 miles each day. We were exhausted and so were the horses!
June 4	We left for Pittsburgh via Philadelphia and Lancaster, traveling by horseback.
June 12	The 400-mile horseback journey to Pittsburgh took 8 days. We covered about 65 miles per day, but we could not travel for two days due to driving rainstorms that washed out the road.



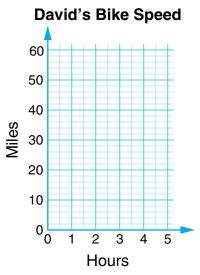


	Time		
			MATH
	-		
3	Complete th	ne table. Decimal	Percent
		0.18	
			37.5%
	<u>45</u> 50		
	<u>16</u> 25		
		0.88	
	earest	Fraction <u> 45</u> 50 <u> 16</u>	earest centimeter? 3. Complete the table. Fraction Decimal 0.18 0.18 $\frac{45}{50}$ 0.18 $\frac{16}{25}$ 0.18

4. Complete the table. Then graph the data and connect the points with line segments. David rides his bike at a speed of about 12 miles per hour.

Rule: 12 * number of hours = total miles

Total Miles
12
36
42



Reference

Equivalent Fractions, Decimals, and Percents

1 1																	
3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 0.3 0.6 66 ³ / ₃ 1 15 16 17 16 17 18 17 10 11 18 12 15 16 0.6 66 ³ / ₃ 1 18 12 16 15 12 21 22 23 23 24 25 <th25< th=""> <th25< th=""> <th25< th=""></th25<></th25<></th25<>	$\frac{1}{2}$	$\frac{2}{4}$	<u>3</u> 6	$\frac{4}{8}$	<u>5</u> 10	<u>6</u> 12	<u>7</u> 14	<u>8</u> 16	<u>9</u> 18	<u>10</u> 20	<u>11</u> 22	12 24	<u>13</u> 26	$\frac{14}{28}$	15 30	0.5	50%
1 2 1 2	$\frac{1}{3}$	<u>2</u> 6	<u>3</u> 9	$\frac{4}{12}$	<u>5</u> 15	<u>6</u> 18	<u>7</u> 21	<u>8</u> 24	<u>9</u> 27	<u>10</u> 30		<u>12</u> 36	<u>13</u> 39	$\frac{14}{42}$	<u>15</u> 45	0.3	33 ¹ / ₃ %
3 8 9 12 <td>$\frac{2}{3}$</td> <td>$\frac{4}{6}$</td> <td><u>6</u> 9</td> <td><u>8</u> 12</td> <td><u>10</u> 15</td> <td><u>12</u> 18</td> <td>14 21</td> <td><u>16</u> 24</td> <td><u>18</u> 27</td> <td>$\frac{20}{30}$</td> <td>22 33</td> <td>$\frac{24}{36}$</td> <td>26 39</td> <td>$\frac{28}{42}$</td> <td>$\frac{30}{45}$</td> <td>0.6</td> <td>$66\frac{2}{3}\%$</td>	$\frac{2}{3}$	$\frac{4}{6}$	<u>6</u> 9	<u>8</u> 12	<u>10</u> 15	<u>12</u> 18	14 21	<u>16</u> 24	<u>18</u> 27	$\frac{20}{30}$	22 33	$\frac{24}{36}$	26 39	$\frac{28}{42}$	$\frac{30}{45}$	0.6	$66\frac{2}{3}\%$
1 $\frac{1}{10}$ $\frac{1}{15}$ $\frac{1}{20}$ $\frac{5}{25}$ $\frac{6}{50}$ $\frac{7}{15}$ $\frac{8}{40}$ $\frac{9}{45}$ $\frac{10}{50}$ $\frac{15}{60}$ $\frac{16}{65}$ $\frac{17}{70}$ $\frac{7}{75}$ 0.2 20% 2 $\frac{1}{10}$ $\frac{1}{5}$ $\frac{2}{20}$ $\frac{1}{25}$ $\frac{1}{25}$ $\frac{1}{26}$ $\frac{1}{26}$ $\frac{2}{66}$ $\frac{2}{66}$ $\frac{2}{66}$ $\frac{2}{67}$ $\frac{2}{75}$ </td <td>$\frac{1}{4}$</td> <td><u>2</u> 8</td> <td><u>3</u> 12</td> <td>$\frac{4}{16}$</td> <td><u>5</u> 20</td> <td><u>6</u> 24</td> <td><u>7</u> 28</td> <td><u>8</u> 32</td> <td><u>9</u> 36</td> <td>$\frac{10}{40}$</td> <td><u>11</u> 44</td> <td>12 48</td> <td><u>13</u> 52</td> <td>14 56</td> <td>15 60</td> <td>0.25</td> <td>25%</td>	$\frac{1}{4}$	<u>2</u> 8	<u>3</u> 12	$\frac{4}{16}$	<u>5</u> 20	<u>6</u> 24	<u>7</u> 28	<u>8</u> 32	<u>9</u> 36	$\frac{10}{40}$	<u>11</u> 44	12 48	<u>13</u> 52	14 56	15 60	0.25	25%
2 40 6 8 10 12 14 14 25 25 26 26 27 37 0.4 40% 3 60 15 12 15 13 14 15 16 37 47 47 30 35 36 38 47 47 60 60% 4 80 12 13 42 50 48 49 45 48 48 45 48 48 45 48	$\frac{3}{4}$	<u>6</u> 8	<u>9</u> 12	<u>12</u> 16	<u>15</u> 20	<u>18</u> 24	21 28	24 32	27 36	$\frac{30}{40}$	33 44	36 48	<u>39</u> 52	42 56	45 60	0.75	75%
$\frac{3}{5}$ $\frac{6}{60}$ $\frac{9}{75}$ $\frac{12}{25}$ $\frac{15}{56}$ $\frac{36}{36}$ $\frac{21}{45}$ $\frac{25}{46}$ $\frac{24}{45}$ $\frac{27}{45}$ $\frac{85}{60}$ $\frac{85}{60}$ $\frac{85}{70}$ $\frac{97}{75}$ 0.6 600% $\frac{4}{5}$ $\frac{8}{10}$ $\frac{12}{5}$ $\frac{12}{20}$ $\frac{25}{20}$ $\frac{23}{20}$ $\frac{33}{20}$ $\frac{45}{20}$ $\frac{25}{20}$ $\frac{25}{20}$ $\frac{25}{20}$ $\frac{23}{20}$ $\frac{23}{20}$ $\frac{23}{20}$ $\frac{33}{20}$ $\frac{45}{20}$ $\frac{25}{20}$ $\frac{25}{20}$ $\frac{25}{20}$ $\frac{23}{20}$ <td< td=""><td>$\frac{1}{5}$</td><td><u>2</u> 10</td><td><u>3</u> 15</td><td>$\frac{4}{20}$</td><td>$\frac{5}{25}$</td><td>$\frac{6}{30}$</td><td>$\frac{7}{35}$</td><td>$\frac{8}{40}$</td><td>$\frac{9}{45}$</td><td></td><td><u>11</u> 55</td><td>12 60</td><td>13 65</td><td><u>14</u> 70</td><td><u>15</u> 75</td><td>0.2</td><td>20%</td></td<>	$\frac{1}{5}$	<u>2</u> 10	<u>3</u> 15	$\frac{4}{20}$	$\frac{5}{25}$	$\frac{6}{30}$	$\frac{7}{35}$	$\frac{8}{40}$	$\frac{9}{45}$		<u>11</u> 55	12 60	13 65	<u>14</u> 70	<u>15</u> 75	0.2	20%
$\frac{1}{5}$ $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{20}$ $\frac{2}{24}$ $\frac{2}{30}$ $\frac{2}{34}$ $\frac{3}{45}$ $\frac{4}{50}$ $\frac{4}{55}$ $\frac{5}{65}$ $\frac{5}{70}$ $\frac{5}{70}$ $\frac{6}{70}$ $\frac{1}{10}$ $\frac{1}{12}$ $\frac{1}{13}$ $\frac{1}{14}$ $\frac{1}{50}$ $\frac{1}{16\frac{3}{3}}$ $\frac{1}{6}$ $\frac{1}{12}$ $\frac{1}{13}$ $\frac{4}{24}$ $\frac{5}{30}$ $\frac{3}{36}$ $\frac{4}{22}$ $\frac{4}{80}$ $\frac{6}{56}$ $\frac{6}{50}$ $\frac{6}{56}$ $\frac{7}{76}$ $\frac{8}{70}$ $\frac{8}{73}$ $\frac{8}{83}$ $\frac{8}{33}$ $\frac{1}{7}$ $\frac{1}{14}$ $\frac{2}{21}$ $\frac{2}{25}$ $\frac{3}{30}$ $\frac{3}{42}$ $\frac{4}{40}$ $\frac{5}{50}$ $\frac{6}{56}$ $\frac{6}{72}$ $\frac{7}{76}$ $\frac{8}{93}$ $\frac{8}{30}$ $\frac{8}{33}$ $\frac{8}{33}$ $\frac{1}{7}$ $\frac{1}{42}$ $\frac{1}{28}$ $\frac{3}{35}$ $\frac{4}{42}$ $\frac{4}{45}$ $\frac{6}{60}$ $\frac{7}{77}$ $\frac{8}{44}$ $\frac{9}{61}$ $\frac{9}{98}$ $\frac{9}{00}$ 0.163 14.3 $\frac{7}{7}$ $\frac{4}{4}$ $\frac{2}{21}$ $\frac{1}{28}$ $\frac{1}{12}$ $\frac{1}{44}$ $\frac{1}{65}$ $\frac{9}{61}$ $\frac{7}{77}$ $\frac{8}{44}$ $\frac{9}{61}$ $\frac{9}{98}$ $\frac{9}{00}$ 0.286 28.6 $\frac{7}{7}$ $\frac{1}{4}$ $\frac{2}{21}$ $\frac{2}{28}$ $\frac{3}{22}$ $\frac{2}{42}$ $\frac{2}{48}$ $\frac{3}{26}$ $\frac{3}{70}$ $\frac{7}{77}$ $\frac{8}{84}$ $\frac{9}{91}$ $\frac{9}{98}$ $\frac{9}{00}$ 0.571 57.1 $\frac{7}{14}$ $\frac{1}{21}$ $\frac{2}{20}$ $\frac{2}{25}$ $\frac{3}{42}$ $\frac{4}{45}$ $\frac{6}{63}$ $\frac{7}{70}$ $\frac{7}{74}$ \frac	2 5	$\frac{4}{10}$	<u>6</u> 15	$\frac{8}{20}$	10 25	$\frac{12}{30}$	14 35	$\frac{16}{40}$	<u>18</u> 45	20 50	22 55	24 60	26 65	28 70	30 75	0.4	40%
$\frac{1}{6}$ $\frac{1}{22}$ $\frac{3}{18}$ $\frac{4}{24}$ $\frac{5}{50}$ $\frac{6}{56}$ $\frac{7}{72}$ $\frac{8}{48}$ $\frac{9}{54}$ $\frac{6}{50}$ $\frac{6}{56}$ $\frac{7}{72}$ $\frac{1}{78}$ $\frac{1}{84}$ $\frac{1}{95}$ $0.1\overline{6}$ $16\frac{2}{3}\%$ $\frac{8}{10}$ $\frac{1}{12}$ $\frac{1}{18}$ $\frac{2}{24}$ $\frac{2}{25}$ $\frac{3}{36}$ $\frac{4}{48}$ $\frac{4}{55}$ $\frac{6}{50}$ $\frac{6}{56}$ $\frac{7}{64}$ $\frac{7}{64}$ $\frac{7}{50}$ $0.8\overline{3}$ $83\frac{1}{3}\%$ $\frac{1}{7}$ $\frac{1}{4}$ $\frac{2}{21}$ $\frac{4}{35}$ $\frac{5}{52}$ $\frac{6}{42}$ $\frac{7}{10}$ $\frac{1}{17}$ $\frac{1}{12}$ $\frac{1}{31}$ $\frac{1}{48}$ $\frac{1}{15}$ 0.143 14.3% $\frac{7}{7}$ $\frac{4}{4}$ $\frac{6}{51}$ $\frac{8}{50}$ $\frac{7}{70}$ $\frac{7}{77}$ $\frac{24}{84}$ $\frac{9}{91}$ $\frac{28}{98}$ $\frac{3}{105}$ 0.286 28.6% $\frac{7}{6}$ $\frac{9}{41}$ $\frac{1}{25}$ $\frac{1}{25}$ $\frac{27}{22}$ $\frac{27}{77}$ $\frac{24}{84}$ $\frac{9}{91}$ $\frac{9}{98}$ $\frac{1}{105}$ 0.249 42.9% $\frac{7}{6}$ $\frac{9}{41}$ $\frac{1}{25}$ $\frac{27}{25}$ $\frac{27}{42}$ $\frac{28}{49}$ $\frac{30}{56}$ $\frac{9}{91}$ $\frac{9}{91}$ $\frac{9}{98}$ $\frac{1}{105}$ 0.2786 28.6% $\frac{7}{7}$ $\frac{1}{14}$ $\frac{1}{21}$ $\frac{28}{29}$ $\frac{30}{25}$ $\frac{42}{21}$ $\frac{48}{29}$ $\frac{50}{57}$ $\frac{57}{77}$ $\frac{84}{84}$ $\frac{91}{91}$ $\frac{91}{75}$ $\frac{91}{75}$ 0.757 0.757 0.757 0.757 0.757 0.757 0.757 0.757 0.757 0.757 <t< td=""><td>$\frac{3}{5}$</td><td><u>6</u> 10</td><td><u>9</u> 15</td><td><u>12</u> 20</td><td><u>15</u> 25</td><td><u>18</u> 30</td><td>21 35</td><td>$\frac{24}{40}$</td><td>27 45</td><td><u>30</u> 50</td><td>33 55</td><td>36 60</td><td>39 65</td><td>$\frac{42}{70}$</td><td><u>45</u> 75</td><td>0.6</td><td>60%</td></t<>	$\frac{3}{5}$	<u>6</u> 10	<u>9</u> 15	<u>12</u> 20	<u>15</u> 25	<u>18</u> 30	21 35	$\frac{24}{40}$	27 45	<u>30</u> 50	33 55	36 60	39 65	$\frac{42}{70}$	<u>45</u> 75	0.6	60%
$\frac{6}{6}$ $\frac{10}{12}$ $\frac{15}{8}$ $\frac{25}{24}$ $\frac{30}{36}$ $\frac{35}{42}$ $\frac{40}{48}$ $\frac{45}{54}$ $\frac{50}{60}$ $\frac{65}{66}$ $\frac{70}{72}$ $\frac{70}{78}$ $\frac{75}{90}$ $0.8\overline{3}$ $83\frac{1}{3}\%$ $\frac{1}{7}$ $\frac{1}{4}$ $\frac{3}{21}$ $\frac{4}{28}$ $\frac{5}{53}$ $\frac{6}{2}$ $\frac{7}{49}$ $\frac{8}{6}$ $\frac{9}{93}$ $\frac{10}{17}$ $\frac{11}{12}$ $\frac{13}{91}$ $\frac{14}{18}$ $\frac{15}{105}$ 14.3% $\frac{2}{7}$ $\frac{4}{14}$ $\frac{6}{21}$ $\frac{8}{28}$ $\frac{10}{33}$ $\frac{12}{42}$ $\frac{14}{49}$ $\frac{16}{16}$ $\frac{8}{8}$ $\frac{20}{77}$ $\frac{24}{24}$ $\frac{26}{91}$ $\frac{28}{98}$ $\frac{30}{105}$ 0.286 28.6% $\frac{7}{14}$ $\frac{9}{21}$ $\frac{12}{28}$ $\frac{13}{15}$ $\frac{14}{49}$ $\frac{24}{49}$ $\frac{26}{61}$ $\frac{27}{77}$ $\frac{24}{84}$ $\frac{26}{91}$ $\frac{28}{98}$ $\frac{30}{105}$ 0.286 28.6% $\frac{7}{14}$ $\frac{12}{12}$ $\frac{13}{25}$ $\frac{14}{49}$ $\frac{24}{49}$ $\frac{26}{65}$ $\frac{27}{77}$ $\frac{24}{84}$ $\frac{26}{91}$ $\frac{28}{79}$ $\frac{70}{79}$ $\frac{77}{78}$ $\frac{84}{91}$ $\frac{97}{95}$ 0.757 0.757 0.757 0.67 0.771 771.4% $\frac{7}{14}$ $\frac{12}{12}$ $\frac{28}{29}$ $\frac{39}{39}$ $\frac{42}{49}$ $\frac{48}{45}$ $\frac{47}{48}$ </td <td>$\frac{4}{5}$</td> <td><u>8</u> 10</td> <td><u>12</u> 15</td> <td>$\frac{16}{20}$</td> <td>20 25</td> <td>$\frac{24}{30}$</td> <td>28 35</td> <td>$\frac{32}{40}$</td> <td>36 45</td> <td></td> <td>44 55</td> <td>$\frac{48}{60}$</td> <td>52 65</td> <td>56 70</td> <td>60 75</td> <td>0.8</td> <td>80%</td>	$\frac{4}{5}$	<u>8</u> 10	<u>12</u> 15	$\frac{16}{20}$	20 25	$\frac{24}{30}$	28 35	$\frac{32}{40}$	36 45		44 55	$\frac{48}{60}$	52 65	56 70	60 75	0.8	80%
17 14 31 48 55 64 7 8 9 10 17 12 13 14 15 0.143 14.3% 27 44 61 88 105 12 14 166 18 20 277 24 261 286 0.286 28.6% 37 61 91 122 155 182 241 246 27 30 37 364 391 42 455 0.429 42.9% 4 81 121 126 205 242 286 323 36 407 477 884 521 566 6105 0.714 71.4% 57 104 125 205 242 285 325 426 455 500 557 600 657 70 77.5 844 991 055 0.714 71.4% 57 124 18 242 305 362 42 485 541 600 657 70 75 0.714 71.4% 6^7 124 18 212 150 48 556 633 70 77 844 991 050 0.857 75.7 8^7 124 18 212 126 426 486 576 621 786 898 901 0.571 71.4% 8^7 81 91 186 111 121 188 121 786 896 9	$\frac{1}{6}$	<u>2</u> 12	<u>3</u> 18	$\frac{4}{24}$	$\frac{5}{30}$	$\frac{6}{36}$	$\frac{7}{42}$	$\frac{8}{48}$	9 54	10 60	<u>11</u> 66	<u>12</u> 72	<u>13</u> 78	14 84	15 90	0.16	16 ² / ₃ %
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	<u>5</u> 6	<u>10</u> 12	<u>15</u> 18	20 24	25 30	<u>30</u> 36	35 42	$\frac{40}{48}$	45 54	50 60	55 66	60 72	65 78	70 84	75 90	0.83	83 ¹ / ₃ %
$\frac{3}{7}$ $\frac{6}{14}$ $\frac{9}{21}$ $\frac{12}{28}$ $\frac{15}{35}$ $\frac{18}{42}$ $\frac{21}{49}$ $\frac{26}{56}$ $\frac{27}{63}$ $\frac{30}{70}$ $\frac{37}{77}$ $\frac{86}{84}$ $\frac{91}{98}$ $\frac{42}{156}$ $\frac{16}{156}$ 0.429 42.9% $\frac{4}{7}$ $\frac{8}{14}$ $\frac{12}{21}$ $\frac{16}{28}$ $\frac{20}{35}$ $\frac{24}{42}$ $\frac{28}{86}$ $\frac{32}{66}$ $\frac{40}{77}$ $\frac{44}{84}$ $\frac{8}{91}$ $\frac{56}{98}$ $\frac{60}{105}$ 0.571 57.1% $\frac{5}{11}$ $\frac{12}{12}$ $\frac{28}{28}$ $\frac{32}{35}$ $\frac{36}{42}$ $\frac{48}{48}$ $\frac{61}{70}$ $\frac{77}{77}$ $\frac{84}{84}$ $\frac{91}{91}$ $\frac{70}{98}$ $\frac{71}{755}$ 0.714 71.4% $\frac{6}{7}$ $\frac{12}{14}$ $\frac{24}{28}$ $\frac{32}{35}$ $\frac{42}{49}$ $\frac{48}{56}$ $\frac{61}{64}$ $\frac{70}{77}$ $\frac{72}{84}$ $\frac{78}{91}$ $\frac{84}{98}$ $\frac{90}{105}$ 0.857 85.7% $\frac{1}{8}$ $\frac{21}{24}$ $\frac{32}{42}$ $\frac{42}{49}$ $\frac{48}{56}$ $\frac{61}{64}$ $\frac{70}{72}$ $\frac{71}{84}$ $\frac{91}{91}$ $\frac{11}{12}$ 15.7 0.125 $12\frac{1}{2}\%$ $\frac{1}{8}$ $\frac{21}{24}$ $\frac{24}{32}$ $\frac{50}{64}$ $\frac{61}{76}$ $\frac{84}{72}$ $\frac{90}{91}$ $\frac{33}{91}$ $\frac{42}{45}$ $\frac{45}{10}$ $\frac{45}{112}$ $\frac{45}{12}$ $\frac{45}{12}$ $\frac{45}{12}$ $\frac{47}{72}$ $\frac{30}{80}$ $\frac{33}{86}$ $\frac{91}{91}$ $\frac{42}{112}$ $\frac{45}{12}$ $\frac{45}{12}$ $\frac{47}{12}$ $\frac{37}{12}$ $\frac{37}{80}$ $\frac{33}{86}$ $\frac{36}{104}$ $\frac{41}{12}$ $\frac{45}{12}$ $\frac{45}{12}$	$\frac{1}{7}$	<u>2</u> 14	3 21	$\frac{4}{28}$	$\frac{5}{35}$	$\frac{6}{42}$	$\frac{7}{49}$	<u>8</u> 56	$\frac{9}{63}$	<u>10</u> 70	<u>11</u> 77	12 84	13 91	14 98	<u>15</u> 105	0.143	14.3%
$\frac{4}{7}$ $\frac{8}{14}$ $\frac{12}{21}$ $\frac{16}{28}$ $\frac{20}{35}$ $\frac{24}{49}$ $\frac{8}{49}$ $\frac{32}{56}$ $\frac{36}{63}$ $\frac{40}{70}$ $\frac{47}{77}$ $\frac{88}{84}$ $\frac{52}{91}$ $\frac{56}{98}$ $\frac{60}{105}$ 0.571 57.1% $\frac{5}{7}$ $\frac{10}{14}$ $\frac{11}{21}$ $\frac{20}{28}$ $\frac{35}{35}$ $\frac{40}{49}$ $\frac{45}{56}$ $\frac{50}{77}$ $\frac{57}{77}$ $\frac{60}{84}$ $\frac{65}{91}$ $\frac{70}{98}$ $\frac{75}{705}$ 0.714 71.4% $\frac{6}{7}$ $\frac{12}{21}$ $\frac{24}{23}$ $\frac{30}{35}$ $\frac{42}{49}$ $\frac{45}{86}$ $\frac{50}{61}$ $\frac{67}{72}$ $\frac{78}{91}$ $\frac{98}{84}$ $\frac{90}{905}$ 0.8577 85.7% $\frac{1}{8}$ $\frac{24}{16}$ $\frac{30}{24}$ $\frac{42}{49}$ $\frac{48}{86}$ $\frac{61}{61}$ $\frac{72}{124}$ $\frac{78}{114}$ $\frac{114}{112}$ $\frac{15}{150}$ 0.125 $12\frac{1}{2}\frac{1}{2}\%$ $\frac{1}{8}$ $\frac{21}{16}$ $\frac{34}{24}$ $\frac{43}{25}$ $\frac{56}{64}$ $\frac{61}{72}$ $\frac{30}{80}$ $\frac{38}{36}$ $\frac{39}{90}$ $\frac{31}{112}$ $\frac{114}{124}$ $\frac{15}{150}$ 0.125 $12\frac{1}{2}\frac{1}{2}\%$ $\frac{3}{8}$ $\frac{61}{64}$ $\frac{92}{22}$ $\frac{26}{20}$ $\frac{30}{20}$ $\frac{32}{32}$ $\frac{36}{26}$ $\frac{37}{2}$ $\frac{81}{80}$ $\frac{91}{100}$ $\frac{98}{112}$ $\frac{91}{105}$ 0.875 $87\frac{1}{2}\%$ $\frac{1}{8}$ $\frac{14}{22}$ $\frac{21}{28}$ $\frac{35}{45}$ $\frac{64}{42}$ $\frac{72}{27}$ $\frac{80}{80}$ $\frac{81}{896}$ $\frac{91}{104}$ $\frac{98}{112}$ 105 0.875 $87\frac{1}{2}\%$ $\frac{1}{8}$ $\frac{14}{22}$ <td>2 7</td> <td>$\frac{4}{14}$</td> <td><u>6</u> 21</td> <td>$\frac{8}{28}$</td> <td><u>10</u> 35</td> <td>$\frac{12}{42}$</td> <td>14 49</td> <td>16 56</td> <td>18 63</td> <td>20 70</td> <td>22 77</td> <td>24 84</td> <td>26 91</td> <td>28 98</td> <td><u>30</u> 105</td> <td>0.286</td> <td>28.6%</td>	2 7	$\frac{4}{14}$	<u>6</u> 21	$\frac{8}{28}$	<u>10</u> 35	$\frac{12}{42}$	14 49	16 56	18 63	20 70	22 77	24 84	26 91	28 98	<u>30</u> 105	0.286	28.6%
57 10 15 20 25 30 34 40 45 50 57 60 65 70 75 0.714 71.4% 67 12 18 24 30 36 42 48 54 60 67 72 78 84 90 0.857 85.7% 18 21 22 35 42 48 54 60 67 72 78 84 90 0.857 85.7% 18 21 22 35 42 48 54 60 67 72 78 84 90 0.125 $122^{1}\%$ 18 21 23 40 42 47 100 118 126 130 114 112 150 0.125 $122^{1}\%$ 38 61 9 122 150 188 21 24 277 300 333 366 39 121 150 0.125 $122^{1}\%$ 38 10 15 202 255 30 355 60 655 70 77 120 755 0.375 $371^{2}\%$ 58 10 15 202 255 30 356 60 655 70 71 112 152 0.375 $371^{2}\%$ 8 10 12 48 56 64 72 80 55 60 65 104 112 150 0.76 0.76 8 <td>$\frac{3}{7}$</td> <td><u>6</u> 14</td> <td><u>9</u> 21</td> <td>12 28</td> <td><u>15</u> 35</td> <td><u>18</u> 42</td> <td>21 49</td> <td>24 56</td> <td>27 63</td> <td><u>30</u> 70</td> <td>33 77</td> <td>36 84</td> <td><u>39</u> 91</td> <td>42 98</td> <td><u>45</u> 105</td> <td>0.429</td> <td>42.9%</td>	$\frac{3}{7}$	<u>6</u> 14	<u>9</u> 21	12 28	<u>15</u> 35	<u>18</u> 42	21 49	24 56	27 63	<u>30</u> 70	33 77	36 84	<u>39</u> 91	42 98	<u>45</u> 105	0.429	42.9%
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $	$\frac{4}{7}$	<u>8</u> 14	12 21	16 28	20 35	$\frac{24}{42}$	$\frac{28}{49}$	32 56	36 63	$\frac{40}{70}$	44 77	48 84	52 91	56 98	<u>60</u> 105	0.571	57.1%
$ \frac{1}{8} \frac{2}{16} \frac{3}{24} \frac{4}{32} \frac{5}{40} \frac{6}{48} \frac{7}{56} \frac{8}{64} \frac{9}{72} \frac{10}{80} \frac{11}{88} \frac{9}{96} \frac{113}{104} \frac{114}{112} \frac{15}{120} 0.125 12\frac{1}{2}\% \\ 3\frac{8}{10} \frac{6}{16} \frac{9}{24} \frac{12}{32} \frac{15}{40} \frac{18}{48} \frac{21}{56} \frac{24}{64} \frac{27}{72} \frac{30}{80} \frac{33}{88} \frac{96}{96} \frac{39}{104} \frac{412}{112} \frac{45}{120} 0.375 37\frac{1}{2}\% \\ \frac{5}{8} \frac{10}{16} \frac{15}{24} \frac{20}{32} \frac{25}{40} \frac{30}{48} \frac{35}{56} \frac{40}{64} \frac{45}{72} \frac{50}{80} \frac{55}{88} \frac{60}{96} \frac{65}{104} \frac{712}{112} \frac{75}{120} 0.625 62\frac{1}{2}\% \\ \frac{7}{8} \frac{14}{16} \frac{21}{24} \frac{28}{32} \frac{35}{40} \frac{42}{48} \frac{49}{56} \frac{56}{64} \frac{63}{72} \frac{70}{80} \frac{77}{88} \frac{84}{96} \frac{91}{104} \frac{98}{112} \frac{105}{120} 0.875 87\frac{1}{2}\% \\ \frac{1}{9} \frac{1}{18} \frac{27}{27} \frac{36}{86} \frac{5}{54} \frac{6}{73} \frac{7}{82} \frac{9}{91} \frac{10}{90} \frac{11}{99} \frac{12}{108} \frac{11}{117} \frac{14}{126} \frac{15}{135} 0.\overline{1} 11\frac{9}{9}\% \\ \frac{9}{104} \frac{1}{9} \frac{1}{22} \frac{30}{135} \frac{1}{19} \frac{11}{9}\% \\ \frac{9}{11} \frac{1}{9} \frac{1}{10} \frac{1}{117} \frac{1}{26} \frac{1}{30} \frac{1}{117} \frac{1}{126} \frac{1}{13} \frac{1}{117} \frac{1}{126} \frac{1}{13} \frac{1}{11} \frac{1}{119}\% \\ \frac{9}{118} \frac{1}{27} \frac{36}{36} \frac{4}{15} \frac{5}{54} \frac{6}{63} \frac{7}{72} \frac{8}{81} \frac{9}{90} \frac{9}{99} \frac{1}{108} \frac{1}{117} \frac{1}{26} \frac{30}{135} 0.\overline{1} 11\frac{9}{9}\% \\ \frac{9}{11} \frac{8}{18} \frac{1}{27} \frac{16}{36} \frac{20}{45} \frac{24}{54} \frac{28}{63} \frac{32}{72} \frac{36}{81} \frac{40}{90} \frac{44}{99} \frac{48}{108} \frac{52}{117} \frac{56}{126} \frac{60}{135} 0.\overline{4} \frac{44\frac{9}{9}\% \\ \frac{9}{19} \frac{1}{18} \frac{1}{27} \frac{26}{36} \frac{61}{35} \frac{1}{54} \frac{4}{63} \frac{5}{72} \frac{61}{81} \frac{9}{90} \frac{55}{99} \frac{60}{108} \frac{65}{117} \frac{70}{126} \frac{75}{135} 0.\overline{5} 55\frac{9}{9}\% \\ \frac{9}{10} \frac{9}{118} \frac{1}{17} \frac{98}{126} \frac{105}{135} 0.\overline{7} 77\frac{9}{9}\% \\ \frac{9}{18} \frac{1}{18} \frac{21}{27} \frac{28}{36} \frac{35}{35} \frac{42}{49} \frac{9}{63} \frac{55}{52} \frac{63}{63} \frac{70}{79} \frac{77}{99} \frac{84}{108} \frac{91}{17} $	5 7	<u>10</u> 14	15 21	20 28	25 35	$\frac{30}{42}$	35 49	40 56	45 63	50 70	55 77	60 84	65 91	70 98	<u>75</u> 105	0.714	71.4%
8 16 24 32 40 48 56 64 72 80 88 96 104 112 120 0.123 122 78 3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 0.375 37½% 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 0.625 62½% 7 14 21 28 35 40 45 50 55 60 65 70 75 0.625 62½% 7 14 21 28 35 42 49 56 63 70 77 84 91 98 105 0.875 87½% 1 21 28 35 64 72 81 90 91 91 91 91 91 91 91 91 91 91 91 91 91 91	<u>6</u> 7	<u>12</u> 14	<u>18</u> 21	24 28	30 35	36 42	42 49	48 56	54 63	60 70	66 77	72 84	78 91	84 98	<u>90</u> 105	0.857	85.7%
$\frac{5}{8}$ $\frac{10}{16}$ $\frac{15}{24}$ $\frac{20}{32}$ $\frac{25}{40}$ $\frac{30}{48}$ $\frac{35}{56}$ $\frac{40}{72}$ $\frac{45}{280}$ $\frac{58}{88}$ $\frac{60}{96}$ $\frac{65}{104}$ $\frac{71}{12}$ $\frac{75}{120}$ 0.625 $62\frac{1}{2}\%$ $\frac{7}{8}$ $\frac{14}{16}$ $\frac{21}{24}$ $\frac{28}{32}$ $\frac{35}{40}$ $\frac{42}{48}$ $\frac{49}{56}$ $\frac{56}{64}$ $\frac{71}{72}$ $\frac{70}{80}$ $\frac{71}{88}$ $\frac{94}{96}$ $\frac{91}{104}$ $\frac{98}{112}$ $\frac{105}{120}$ 0.875 $87\frac{1}{2}\%$ $\frac{1}{9}$ $\frac{2}{18}$ $\frac{3}{27}$ $\frac{4}{36}$ $\frac{5}{54}$ $\frac{6}{63}$ $\frac{7}{72}$ $\frac{81}{80}$ $\frac{91}{99}$ $\frac{98}{112}$ $\frac{105}{120}$ 0.875 $87\frac{1}{2}\%$ $\frac{1}{9}$ $\frac{2}{18}$ $\frac{3}{27}$ $\frac{4}{46}$ $\frac{5}{54}$ $\frac{6}{63}$ $\frac{7}{72}$ $\frac{81}{90}$ $\frac{91}{99}$ $\frac{98}{112}$ $\frac{105}{120}$ 0.875 $87\frac{1}{2}\%$ $\frac{2}{9}$ $\frac{4}{48}$ $\frac{5}{54}$ $\frac{6}{63}$ $\frac{7}{72}$ $\frac{8}{81}$ $\frac{91}{90}$ $\frac{91}{112}$ $\frac{13}{117}$ $\frac{14}{126}$ $\frac{15}{135}$ $0.\overline{1}$ $11\frac{9}{19}\%$ $\frac{2}{9}$ $\frac{4}{48}$ $\frac{6}{27}$ $\frac{8}{36}$ $\frac{10}{145}$ $\frac{12}{14}$ $\frac{16}{12}$ $\frac{18}{18}$ 20 22 $\frac{24}{108}$ $\frac{26}{216}$ $\frac{28}{28}$ $\frac{30}{30}$ $0.\overline{2}$ $22\frac{9}{9}\%$ $\frac{4}{9}$ $\frac{8}{18}$ $\frac{12}{27}$ $\frac{16}{36}$ $\frac{20}{24}$ $\frac{28}{263}$ $\frac{32}{22}$ $\frac{36}{90}$ $\frac{49}{99}$ $\frac{48}{108}$ $\frac{52}{117}$ $\frac{56}{126}$ $\frac{60}{135}$ <td>$\frac{1}{8}$</td> <td><u>2</u> 16</td> <td>$\frac{3}{24}$</td> <td>$\frac{4}{32}$</td> <td>$\frac{5}{40}$</td> <td>$\frac{6}{48}$</td> <td><u>7</u> 56</td> <td>$\frac{8}{64}$</td> <td>$\frac{9}{72}$</td> <td>$\frac{10}{80}$</td> <td><u>11</u> 88</td> <td>12 96</td> <td><u>13</u> 104</td> <td><u>14</u> 112</td> <td><u>15</u> 120</td> <td>0.125</td> <td>$12\frac{1}{2}\%$</td>	$\frac{1}{8}$	<u>2</u> 16	$\frac{3}{24}$	$\frac{4}{32}$	$\frac{5}{40}$	$\frac{6}{48}$	<u>7</u> 56	$\frac{8}{64}$	$\frac{9}{72}$	$\frac{10}{80}$	<u>11</u> 88	12 96	<u>13</u> 104	<u>14</u> 112	<u>15</u> 120	0.125	$12\frac{1}{2}\%$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	<u>3</u> 8	<u>6</u> 16	9 24	12 32	15 40	18 48	21 56	24 64	27 72	30 80	33 88	36 96	<u>39</u> 104	<u>42</u> 112	$\frac{45}{120}$	0.375	$37\frac{1}{2}\%$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	<u>5</u> 8	<u>10</u> 16	<u>15</u> 24	$\frac{20}{32}$	25 40	$\frac{30}{48}$	35 56	$\frac{40}{64}$	45 72	50 80	55 88	60 96	<u>65</u> 104	<u>70</u> 112	<u>75</u> 120	0.625	$62\frac{1}{2}\%$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	<u>7</u> 8	<u>14</u> 16	21 24	28 32	$\frac{35}{40}$	$\frac{42}{48}$	49 56	56 64	63 72	70 80	77 88	84 96	<u>91</u> 104	<u>98</u> 112	<u>105</u> 120	0.875	87 ¹ / ₂ %
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	<u>1</u> 9	<u>2</u> 18	$\frac{3}{27}$	$\frac{4}{36}$	<u>5</u> 45	$\frac{6}{54}$	$\frac{7}{63}$	<u>8</u> 72	<u>9</u> 81	<u>10</u> 90	<u>11</u> 99	<u>12</u> 108	<u>13</u> 117	<u>14</u> 126	<u>15</u> 135	0.1	11 ¹ / ₉ %
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	<u>2</u> 9	<u>4</u> 18	<u>6</u> 27	<u>8</u> 36	<u>10</u> 45	<u>12</u> 54	<u>14</u> 63	<u>16</u> 72	<u>18</u> 81	20 90	22 99	<u>24</u> 108	<u>26</u> 117	<u>28</u> 126	<u>30</u> 135	0.2	22 ² / ₉ %
$\frac{7}{9} \frac{14}{18} \frac{21}{27} \frac{28}{36} \frac{35}{45} \frac{42}{54} \frac{49}{63} \frac{56}{72} \frac{63}{81} \frac{70}{90} \frac{77}{99} \frac{84}{108} \frac{91}{117} \frac{98}{126} \frac{105}{135} 0.\overline{7} 77^{\frac{7}{9}} \mathbf{\%}$	$\frac{4}{9}$	<u>8</u> 18	$\frac{12}{27}$	$\frac{16}{36}$	$\frac{20}{45}$	24 54	28 63	32 72	36 81	$\frac{40}{90}$	44 99	<u>48</u> 108	<u>52</u> 117	<u>56</u> 126	<u>60</u> 135	0.4	$44\frac{4}{9}\%$
	<u>5</u> 9	<u>10</u> 18	<u>15</u> 27	$\frac{20}{36}$	25 45	30 54	35 63	40 72	45 81	50 90	55 99	<u>60</u> 108	<u>65</u> 117	<u>70</u> 126	<u>75</u> 135	0.5	55 ⁵ 9%
$\frac{8}{9} \frac{16}{18} \frac{24}{27} \frac{32}{36} \frac{40}{45} \frac{48}{54} \frac{56}{63} \frac{64}{72} \frac{72}{81} \frac{80}{90} \frac{88}{99} \frac{96}{108} \frac{104}{117} \frac{112}{126} \frac{120}{135} 0.\overline{8} 88 \frac{8}{9}\%$	<u>7</u> 9	<u>14</u> 18	2 <u>1</u> 27	28 36	<u>35</u> 45	42 54	49 63	56 72	63 81	70 90	7 <u>7</u> 99	<u>84</u> 108	<u>91</u> 117	<u>98</u> 126	<u>105</u> 135	0.7	77 ⁷ 9%
	<u>8</u> 9	<u>16</u> 18	24 27	$\frac{32}{36}$	$\frac{40}{45}$	48 54	56 63	64 72	72 81	80 90	88 99	<u>96</u> 108	<u>104</u> 117	<u>112</u> 126	<u>120</u> 135	0.8	88 ⁸ 9%

Note: The decimals for sevenths have been rounded to the nearest thousandth.

Reference

Metric System

Units of Length		
1 kilometer (km)	=	1000 meters (m)
1 meter	=	10 decimeters (dm)
	=	100 centimeters (cm)
	=	1000 millimeters (mm)
1 decimeter	=	10 centimeters
1 centimeter	=	10 millimeters
Units of Area		
1 square meter (m ²)	=	100 square decimeters (dm ²)
,		10,000 square
		centimeters (cm ²)
1 square decimeter	=	100 square centimeters
1 are (a)		100 square meters
1 hectare (ha)	=	100 ares
1 square kilometer (km ²)	=	100 hectares
Units of Volume		
1 cubic meter (m ³)	=	1000 cubic decimeters (dm ³)
	=	1,000,000 cubic
		centimeters (cm ³)
1 cubic decimeter	=	1000 cubic centimeters
Units of Capacity		
1 kiloliter (kL)	=	1000 liters (L)
1 liter	=	1000 milliliters (mL)
Units of Mass		
1 metric ton (t)	=	1000 kilograms (kg)
1 kilogram	=	1000 grams (g)
1 gram		1000 milligrams (mg)
i yiaili	_	rooo miliigiams (mg)

Units of Time

1 century	=	100 years
1 decade	=	10 years
1 year (yr)	=	12 months
	=	52 weeks (plus one or two days)
	=	365 days (366 days in a leap year)
1 month (mo)	=	28, 29, 30, or 31 days
1 week (wk)	=	7 days
1 day (d)	=	24 hours
1 hour (hr)	=	60 minutes
1 minute (min)	=	60 seconds (sec)

U.S. Customary System

Units of Length	
1 mile (mi)	= 1760 yards (yd)
	= 5280 feet (ft)
1 yard	= 3 feet
	= 36 inches (in.)
1 foot	= 12 inches
Units of Area	
1 square yard (yd ²)	= 9 square feet (ft ²)
	= 1296 square inches (in. ²)
1 square foot	= 144 square inches
1 acre	= 43,560 square feet
1 square mile (mi ²)	= 640 acres
Units of Volume	
1 cubic yard (yd ³)	= 27 cubic feet (ft ³)
1 cubic foot	= 1728 cubic inches (in. 3)
Units of Capacity	
1 gallon (gal)	= 4 quarts (qt)
1 quart	= 2 pints (pt)
1 pint	= 2 cups (c)
1 cup	= 8 fluid ounces (fl oz)
1 fluid ounce	= 2 tablespoons (tbs)
1 tablespoon	= 3 teaspoons (tsp)
Units of Weight	
1 ton (T)	= 2000 pounds (lb)
1 pound	= 16 ounces (oz)

System Equivalents

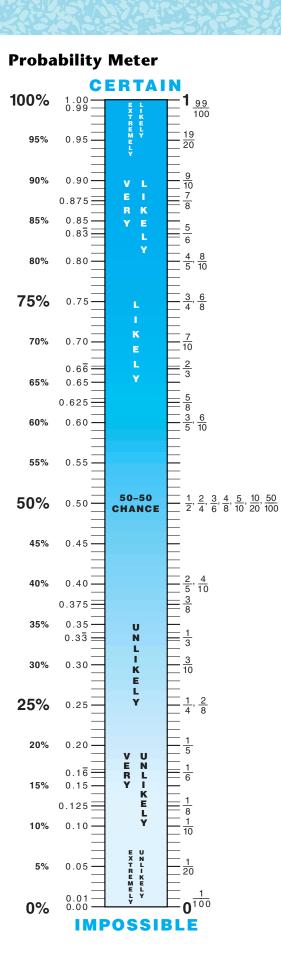
1 inch is about 2.5 cm (2.54)
1 kilometer is about 0.6 mile (0.621)
1 mile is about 1.6 kilometers (1.609)
1 meter is about 39 inches (39.37)
1 liter is about 1.1 quarts (1.057)
1 ounce is about 28 grams (28.350)
1 kilogram is about 2.2 pounds (2.205)
1 hectare is about 2.5 acres (2.47)

Rules for Order of Operations

- 1. Do operations within parentheses or other grouping symbols before doing anything else.
- 2. Calculate all powers.
- **3.** Do multiplications or divisions in order, from left to right.
- **4.** Then do additions or subtractions in order, from left to right.

Place-Value Chart

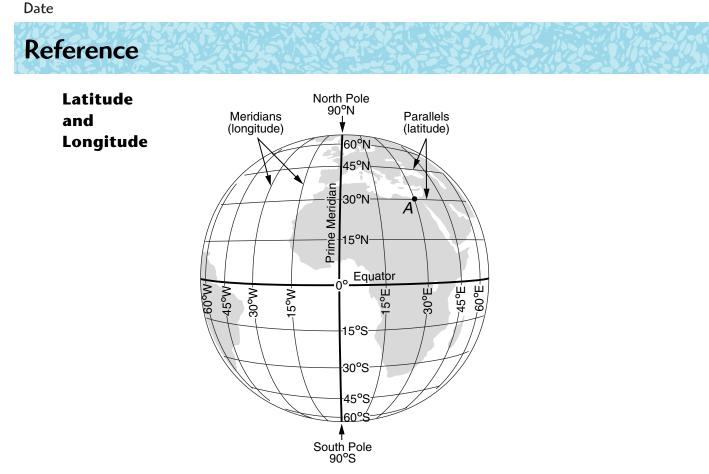
ĩ	100B	10B	crillions 100B 10B billions 100M 10M	100M	10M	millions	hundred- thousands	ten- thousands	thousands	hundreds	tens	ones	-	tenths	thousands hundreds tens ones . tenths hundredths thousandths	thousandths
1000 billions			1000 millions			1,000,000s	100,000s	10,000s	1000s	100s	10s	10s 1s	•	. 0.1s	0.01s	0.001s
~	10 ¹¹	10 ¹⁰	10 ¹² 10 ¹¹ 10 ¹⁰ 10 ⁹ 10 ⁸		10 ⁷	10 ⁶	10 ⁵	104	10 ³	10^2 10^1 10^0 10^{-1}	10 ¹	10 ⁰	•	10 ⁻¹	10 ⁻²	10 ⁻³



Symbols

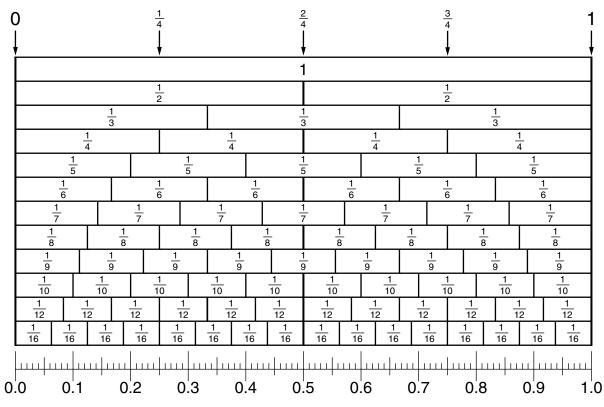
+	plus or positive			
_	minus or negative			
*, ×	multiplied by			
÷,/	divided by			
=	is equal to			
¥	is not equal to			
<	is less than			
>	is greater than			
\leq	is less than or			
	equal to			
≥	is greater than			
	or equal to			
x ⁿ	<i>n</i> th power of <i>x</i>			
\sqrt{x}	square root of x			
%	percent			
a:b, a / b, a	ratio of <i>a</i> to <i>b</i>			
	or a divided by b			
	or the fraction $\frac{a}{b}$			
o	degree			
(<i>a,b</i>)	ordered pair			
ĂŜ	line AS			
AS	line segment AS			
\overrightarrow{AS}	ray <i>AS</i>			
Ŀ.	right angle			
\perp	is perpendicular to			
II	is parallel to			
$\triangle ABC$	triangle ABC			
∠ ABC	angle ABC			
∠ B	angle <i>B</i>			

Reference



Point A is located at 30° N latitude and 30° E longitude.

Fraction-Stick and Decimal Number-Line Chart



Reference

THIS BOOK IS THE PROP	not wr	S to whom this ite on any page	e or mark any	part of it in		
STATE				ay, consumable achers should s		
PROVINCE				clearly written in		•
COUNTY	Book No.			ued.		and in
PARISH	Enter inform	nation		e following term ording the cond		
SCHOOL DISTRICT	in spaces to	o the		od; Fair; Poor;		
OTHER	left as instru	ucted.				
ISSUED TO Year Used	CONDI	ITION				
	ISSUED	RETURNED		"Easy"		
				Fractions	Decimals	Percents
				$\frac{1}{2}$	0.50	50%
				$\frac{1}{4}$	0.25	25%
0 0.125 0.25 0.375 0.5 0.625 0.	75 0.875	1		<u>3</u> 4	0.75	75%
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	<u> </u> 3 <u>7</u> 4 8	$\frac{2}{2}$		1 5	0.20	20%
	4 8 6 8	2 <u>4</u>		2 5	0.40	40%
	Ē			3 5	0.60	60%
$\frac{0}{8}$ $\frac{4}{8}$		<u>8</u> 8		4 5	0.80	80%
Multiplication/Division Facts Table				<u>1</u> 10	0.10	10%
*,/ 1 2 3 4 5 6 7 8 9 10				<u>3</u> 10	0.30	30%

*,/	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Prefixes

trithree	nonanine	hectohundred
quadfour	decaten	decitenth
pentafive	dodecatwelve	centihundredth
hexasix	gigabillion	millithousandth
heptaseven	megamillion	micromillionth
octaeight	kilo thousand	nanobillionth

<u>7</u> 10

<u>9</u> 10

0.70

0.90

70%

90%

Place-Value Chart

millions	hundred- thousands	ten- thousands	thousands	hundreds	tens	ones	tenths	hundredths	thousandths
1,000,000s	100,000s	10,000s	1000s	100s	10s	1s	0.1s	0.01s	0.001s
10 ⁶	10 ⁵	10 ⁴	10 ³	10 ²	10 ¹	10 ⁰	10 ⁻¹	10-2	10 ⁻³

Metric System

Units of Length				U
1 kilometer (km)	=	1000 meters (m)		1
1 meter	=	10 decimeters (dm)		
	=	100 centimeters (cm)		1
		1000 millimeters (mm)		
1 decimeter		10 centimeters		1
1 centimeter	=	10 millimeters		U
Units of Area				1
1 square meter (m ²)	=	100 square decimeters (dm ²)		•
		10,000 square		1
		centimeters (cm ²)		1
1 square decimeter	=	100 square centimeters		1
1 are (a)	=	100 square meters		
1 hectare (ha)		100 ares		U 1
1 square kilometer (km ²)	=	100 hectares		1
Units of Volume				1
1 cubic meter (m ³)	=	1000 cubic decimeters (dm ³)		U
		1,000,000 cubic		1
		centimeters (cm ³)		1
1 cubic decimeter	=	1000 cubic centimeters		1
				1
Units of Capacity	_	1000 liters(1)		1
1 kiloliter (kL) 1 liter		1000 liters (L) 1000 milliliters (mL)		1
i iitei	_			U
Units of Mass				1
1 metric ton (t)	=	1000 kilograms (kg)		1
1 kilogram		1000 grams (g)	L	
1 gram	=	1000 milligrams (mg)		

Units of Time

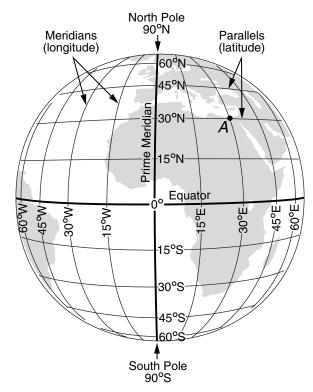
1 century 1 decade		100 years 10 years
1 year (yr)		12 months
	=	52 weeks (plus one or two days)
	=	365 days (366 days in a leap year)
1 month (mo)	=	28, 29, 30, or 31 days
1 week (wk)	=	7 days
1 day (d)	=	24 hours
1 hour (hr)	=	60 minutes
1 minute (min)	=	60 seconds (sec)

System Equivalents

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- 1 liter is about 1.1 quarts (1.057)
- 1 ounce is about 28 grams (28.350)
- 1 kilogram is about 2.2 pounds (2.205)
- 1 hectare is about 2.5 acres (2.47)

U.S. Customary System

		t s of Length le (mi)		1760 yards (yd) 5280 feet (ft)
	1 ya	rd	=	3 feet 36 inches (in.)
	1 foo	ot	=	12 inches
²)	1 sq 1 sq 1 ac	uare foot	= = =	9 square feet (ft ²) 1296 square inches (in. ²) 144 square inches 43,560 square feet 640 acres
	1 cu			27 cubic feet (ft ³) 1728 cubic inches (in. ³)
³)	1 ga 1 qu 1 pir 1 cu 1 flu	nt p id ounce	= = =	4 quarts (qt) 2 pints (pt) 2 cups (c) 8 fluid ounces (fl oz) 2 tablespoons (tbs) 3 teaspoons (tsp)
	Unit 1 tor 1 po	· · /		2000 pounds (lb) 16 ounces (oz)



Point *A* is located at 30° N latitude and 30° E longitude.

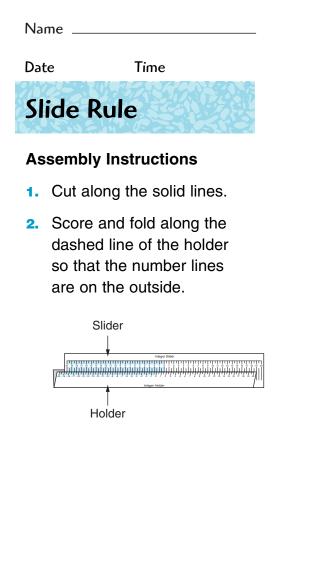
Date

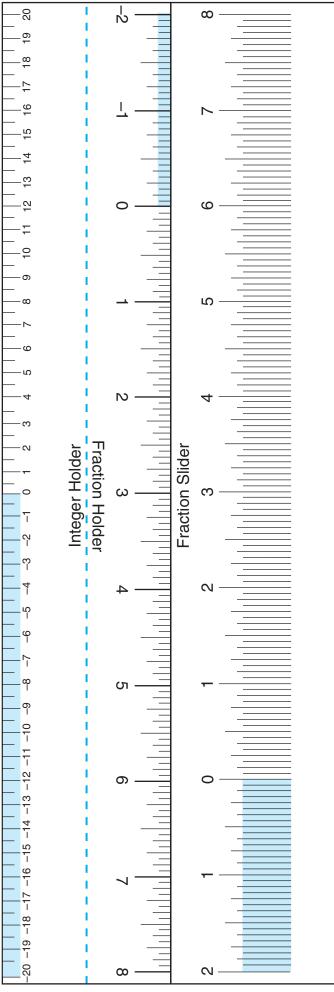
Time

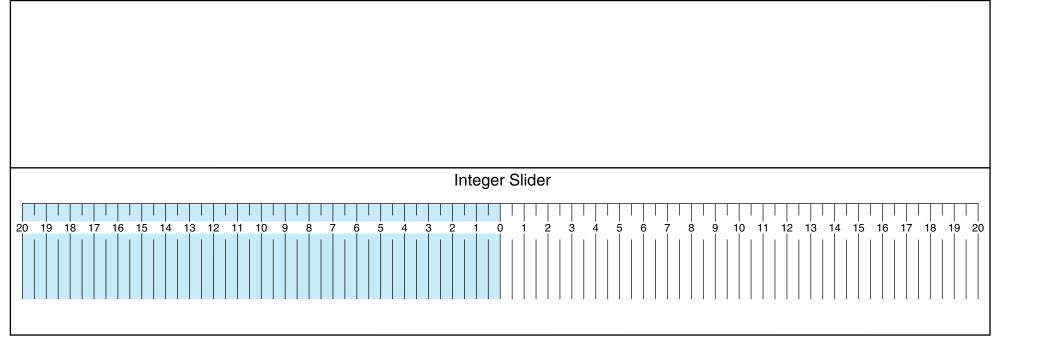
Polygon Capture Property Cards (Front)

There is only one right angle.	There are one or more right angles.	All angles are right angles.	There are no right angles.
There is at least one acute angle.	At least one angle is more than 90°.	All angles are right angles.	There are no right angles.
All opposite sides are parallel.	Only one pair of sides is parallel.	There are no parallel sides.	All sides are the same length.
All opposite sides are parallel.	Some sides have the same length.	All opposite sides have the same length.	Wild Card: Pick your own side property.

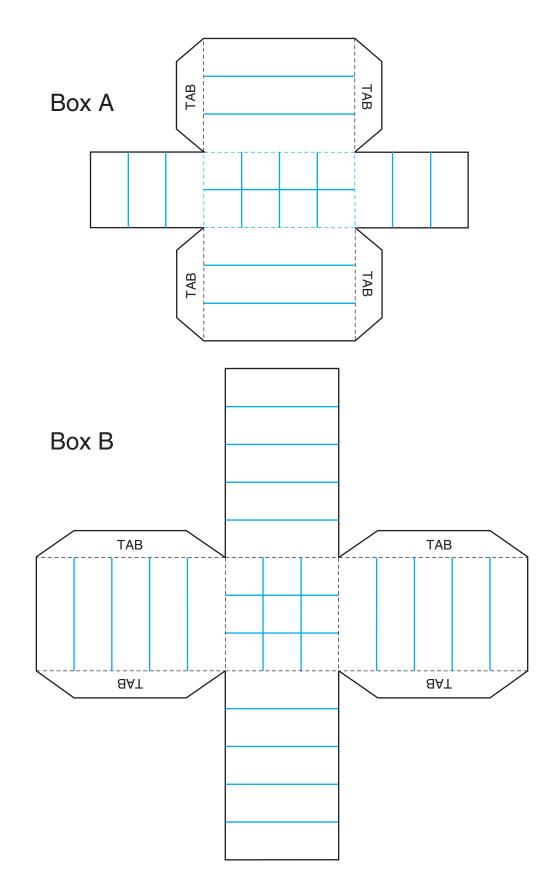
Angles	Angles	Angles	Angles
Angles	Angles	Angles	Angles
Sides	Sides	Sides	Sides
Sides	Sides	Sides	Sides







Rectangular Prism Patterns



Square Tiles